SV

TRANSVERSE VIBRATION OF TRIANGULAR PLATE WITH ARBITRARY THICKNESS VARIATION AND VARIOUS BOUNDARY CONDITIONS

B. SINGH

Department of Mathematics, University of Roorkee, Roorkee, 247667 (U.P.), India

AND

S. M. HASSAN

Department of Mathematics, Faculty of Science, Ain Shams University, Cairo, Egypt

(Received 21 March 1997, and in final form 14 January 1998)

The Rayleigh–Ritz method has been employed to obtain the numerical solution of the vibration problem of a triangular plate with arbitrary thickness variation and various boundary conditions at the three edges. The thickness has been approximated by a polynomial in natural co-ordinates which have been used everywhere as they greatly simplify the calculations. Successive approximations have been worked out until the first three frequencies and mode shapes converge to at least three significant figures. The results are tabulated for selected cases and are compared with known results for uniform and linear thickness variation. Three-dimensional mode shapes have been drawn using the tools for computer graphics.

© 1998 Academic Press Limited

1. INTRODUCTION

The triangular plates constitute an important part of engineering design. It is, therefore, necessary to know beforehand the frequencies and mode shapes of such plates under different conditions. This is the reason why a lot of information already exists in the literature about triangular plates of different shapes, sizes, thickness variations and the conditions at the edges. But our survey reveals that most of the work has been confined to plates of special shapes such as right-angled, isosceles or equilateral triangles. As far as boundary conditions are concerned, a very large number of papers discuss the cantilever plates only. Again most of the papers deal with plates of uniform thickness. The methods range from experimental to approximate and to purely numerical.

Only recently did a few papers appear on triangular plates of linearly varying thickness. Mirza and Bijlani [1] have given some results for cantilever plates with variable thickness. Singh and Saxena [2] have taken a general triangular plate with linearly varying thickness and with various combinations of clamped, simply-supported and free edges. It contains a wealth of information about triangular plates up to 1995. Extensive tables are given for comparison with the earlier results. Prior to this Singh and Chakraverty [3] studied the most general triangular plates of uniform thickness using boundary characteristic orthogonal polynomials in two variables. Such polynomials have



interesting properties which remove numerical instability. In both references [2] and [3] the general triangle was first mapped into a standard right-angled triangle on which the solution is generated. The basis functions were so chosen that the essential boundary conditions are satisfied. Other papers which deal with triangular plates of uniform thickness are Gorman [4-8] who discusses right-angled triangular plates, Bhat [9] who studied polygonal plates in general but gives results for isosceles or right-angled plates, Kim and Dickinson [10] and Lam et al. [11] who gives some results for right-angled isotropic and orthotropic plates. A few more references related with triangular plates are Strand [12], Christensen [13], Gustafson et al. [14], Kuttler and Sigillito [15] and Cowper et al. [16]. References [2] and [3] give detailed comparisons of the results given by these authors. Two more papers have been brought to the notice of the authors. One is Liew [17] which deals with the response of plates of arbitrary shape subject to static loading. The other one is Liew et al. [18] which investigates the flexural vibration of triangular composite plates influenced by fibre orientation. Although these papers have no direct relevance to the present work, these may be of interest to those readers who wish to extend the present work to composite plates of variable thickness.

The basic aim of the present investigation is to study the problem in its most general form i.e., by (1) taking a general triangle, (2) taking an arbitrary thickness variation, and (3) using different combinations of boundary conditions at the three edges of the plate. It is an extension of reference [2] in two ways. First, the linear thickness variation considered in reference [2] has been generalized to a polynomial variation of arbitrary degree. Second, in place of the Cartesian co-ordinates the natural co-ordinates [19, 20] have been used. This not only simplifies the theoretical discussion but gives simple closed form expressions for the integrals involved. A single computer program, by choosing the parameters properly, gives results for virtually any plate.

A sufficiently large number of approximations have been worked out to ensure convergence. The basis functions are taken so that the essential boundary conditions are satisfied. Comparisons have been made with known results in special cases. Tables are given for frequencies and three-dimensional plots are given for mode shapes in some selected cases.

2. METHOD OF SOLUTION

Let the plate occupy the domain R of an xy-plane with vertices numbered 1, 2 and 3 and having co-ordinates as $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (a, 0)$ and $(x_3, y_3) = (b, c)$ as shown



Figure 1. The triangular plate. Shaded area = A_1 , area of $\triangle 123 = A$, $L_1 = A_1/A$.

in Figure 1. Thus, the numbers a, b and c determine the shape of the triangle completely. The Cartesian co-ordinates (x, y) and the natural co-ordinates (L_1, L_2, L_3) of a point P inside the triangle are related by:

$$x = \sum_{i=1}^{3} L_i x_i, \qquad y = \sum_{i=1}^{3} L_i y_i, \qquad (1)$$

and

$$L_i = (\alpha_i + \beta_i x + \gamma_i y)/(2A)$$
⁽²⁾

where α_i , β_i and γ_i are constants and A is the area of the plate. All these can be expressed in terms of the co-ordinates of the vertices. Note that out of L_1 , L_2 and L_3 , only two are linearly independent since $L_1 + L_2 + L_3 = 1$. It is known that

$$\iint_{R} L_{1}^{i} L_{2}^{j} L_{3}^{k} \, \mathrm{d}x \, \mathrm{d}y = \frac{i l j ! k !}{(i + j + k + 2)!} \, (2A). \tag{3}$$

This helps in expressing integrals of polynomials in L_1 , L_2 and L_3 in closed form. Let the variable thickness of the plate be expressed as

$$h = ah_o f(L_1, L_2), \tag{4}$$

where h_o is non-dimensional thickness at some standard point and f is a non-dimensional function of L_1 and L_2 . As we shall see later, the analysis becomes simpler if f is a polynomial. In fact f can always be approximated by a polynomial by measuring the thickness at a suitably chosen set of points and get the interpolating polynomial of the form

$$f \simeq f_M (L_1, L_2) = \sum_{i=1}^{M} a_i L_1^{m_i} L_2^{n_i},$$
(5)

where M is the number of distinct sample points. The constants a_i will depend upon the location of the sample points and m_i , n_i are non-negative itegers. The first ten values of m_i and n_i are as follows:

i	1	2	3	4	5	6	7	8	9	10,
m_i	0	1	0	2	1	0	3	2	1	0,
n_i	0	0	1	0	1	2	0	1	2	3.

The cases of uniform, linear, quadratic and cubic thickness variations correspond to M = 1, 3, 6 and 10, respectively. The constants a_i can be found by measuring thickness at a selected set of points of the plate as already explained. Thus, the function $f_M(L_1, L_2)$ is completely known.

Now, the Rayleigh-Ritz method minimizes

$$\omega^{2} = \frac{\iint_{R} D[[\nabla^{2}W]^{2} + 2(1-v) [W_{xy}^{2} - W_{xx} W_{yy}]] dx dy}{\iint_{R} \rho h W^{2} dx dy},$$
(6)

where

$$D = Eh^3/[12(1 - v^2)] =$$
 flexural rigidity

with E, ρ , v, ω and W as Young's modulus, density, Poisson's ratio, frequency and maximum displacement at (x, y), respectively. In the *N*-term approximation, we take the displacement of the form

$$W = \sum_{i=1}^{N} c_i \phi_i = L_1^{p_1} L_2^{p_2} L_3^{p_3} \sum_{i=1}^{N} c_i L_1^{m_i} L_2^{n_i},$$
(7)

where c_i are constants and ϕ_i are the basis functions which are so chosen that the essential boundary conditions are satisfied. For this we take $p_i = 0$, 1 or 2 accordingly as the edge facing vertex *i* is free, simply-supported or clamped.

Substituting the expressions for h from equation (4) and W from equation (7) in equation (6), one gets after lengthy but straightforward calculations,

$$\sum_{j=1}^{N} (a_{ij} - \lambda^2 b_{ij}) c_j = 0, \qquad i = 1, \dots, N,$$
(8)

where

$$\lambda^{2} = 12(1 - v^{2})\rho a^{2}\omega^{2}/(Eh_{o}^{2}), \qquad (9)$$

and a_{ij} , b_{ij} are given by

$$a_{ij} = \iint_{\mathcal{R}} f^{3}[A_{1} \phi_{i}^{11} \phi_{j}^{11} + A_{2} (\phi_{i}^{11} \phi_{j}^{22} + \phi_{i}^{22} \phi_{j}^{11}) + A_{3} (\phi_{i}^{11} \phi_{j}^{12} + \phi_{i}^{12} \phi_{j}^{12}) + A_{4} \phi_{i}^{12} \phi_{j}^{12} + A_{5} (\phi_{i}^{22} \phi_{j}^{12} + \phi_{i}^{12} \phi_{j}^{22}) + A_{6} \phi_{i}^{22} \phi_{j}^{22}] dx dy,$$
(10)

$$b_{ij} = \iint_{R} f\phi_i \,\phi_j \,\mathrm{d}x \,\mathrm{d}y. \tag{11}$$

Here the superscripts 1 and 2 are used for differentiation with respect to L_1 and L_2 , respectively. The coefficients A_1 through A_6 are given by

$$A_{1} = K_{1}^{2}, \quad A_{2} = K_{1} K_{2} - K_{3}, \quad A_{3} = -2K_{1} K_{4}, \quad A_{4} = 4K_{4}^{2} + 2K_{3},$$
$$A_{5} = -2K_{2} K_{4}, \quad A_{6} = K_{2}^{2}, \quad (12)$$

where

$$\mathbf{K}_1 = 1 + [(\xi - 1)/\eta]^2, \quad \mathbf{K}_2 = 1 + (\xi/\eta)^2, \quad \mathbf{K}_3 = (1 - \nu)/\eta^2,$$

 $K_4 = K_2 - \xi/\eta^2,$

with non-dimensional parameters ξ and η defined by

$$\xi = b/a, \qquad \eta = c/a. \tag{13}$$

Substituting expressions for f, ϕ_i and ϕ_j in equations (10) and (11), one gets a fairly lengthy expression which involves the variables L_1 and L_2 . The choice of thickness and the shape functions as polynomials leads to expressions involving polynomials only. These could be integrated in closed form using equation (3). Thus, expressions for a_{ij} and b_{ij} are available in closed form and can be computed numerically.

System (8) is the standard generalized eigenvalue problem. It has been solved by the Generalized Jacobi method discussed in Wilkinson [21] and Bathe and Wilson [22]. This gives the frequency parameter λ . The associated mode shapes are known from the eigenvector

$$c = [c_1, c_2, \ldots, c_N]^T.$$

3. NUMERICAL WORK AND DISCUSSION

Due to the involvement of a very large number of parameters, it would be a gigantic task to make a detailed study of their effects on the frequencies and mode shapes. The authors, therefore, studied the problem only for a few selected cases. The parameters which have been varied are those which take care of boundary conditions, shape of the plate and thickness variation. The Poisson's ratio has been chosen to be 0.3 because it is this value for which most of the results are available in the literature. In some cases, however, results are given for other values also.

The order of approximation N has been varied from 1 to 36. This is sufficient for convergence of the first three frequencies up to at least three significant figures. All the computations are carried out in double precision arithmetic to avoid numerical instability. The results reported in the tables correspond to N = 36.

3.1. GEOMETRY OF THE PLATE

As already explained, this is controlled by parameters ξ and η . We have examined the plates of the following three shapes: (1) equilateral triangle, $\xi = 0.5$, $\eta = \sqrt{3/2}$ (Figure 2(a)); (2) isosceles right-angled triangle, $\xi = 0.0$, $\eta = 1.0$ (Figure 2(b)); and (3) obtuse isosceles triangle with angles 30, 30 and 120° , $\xi = -0.5$, $\eta = \sqrt{3/2}$ (Figure 2(c)).



Figure 2. (a) Equilateral triangle. (b) Isosceles right-angled triangle. (c) Obtuse isosceles triangle with angles 30, 30 and 120° .

3.2. BOUNDARY CONDITIONS

All combinations of clamped, simply-supported and free boundary have been considered. Thus, p_1 , p_2 and p_3 are given the values 0, 1 or 2. This leads to 27 different combinations in general for a given thickness variation. However, symmetry in shape may reduce this number.

3.3. THICKNESS VARIATION

As already mentioned, M parameters a_1 through a_M control the thickness variation. Numerical work has been carried out for M = 1, 3, 6 and 10 which corresponds to uniform, linear, quadratic and cubic variations, respectively. Thus, up to ten parameters can be suitably varied and a large variety of thickness variations can be examined. The following special cases have been investigated in detail.

3.3.1. Uniform thickness variation

In this case M = 1 and f = 1 (Figure 3(a)). This case has been discussed extensively in the literature with a variety of shapes and boundary conditions at the edges. Complete up-to-date information about this case is available in reference [2]. The results for uniform thickness follow, as a special case, from those of quadratic variation which are reported in Table 2. In this case the results agree completely with reference [2] at N = 28 and the comparison will not be duplicated here.

3.3.2. Linear thickness variation

Here, M = 3. In particular, the authors have considered

$$f = \beta' + (1 - \beta')L_1 + (\alpha' - \beta')L_2.$$
 (Figure 3(b)). (14)

This amounts to assigning values 1, α' and β' to f at vertices 1, 2 and 3, respectively. Similar studies have been made in reference [2]. Reference [1] also gives some results in this case.



Figure 3. (a) Triangular plate with uniform thickness variation. (b) Triangular plate with linear thickness variation. (c, d) Triangular plate with quadratic thickness variation. (e) Triangular plate with cubic thickness variation.

- ~			יי איש ע	$= 0.5, \eta = $	/3/2		$\xi = 0.0, \eta = 1$	ọ.	۳ ۳	$= -0.5, \eta = \sqrt{2}$	/3/2
1 ന	ø	β	, 71	λ_{2}	λ_3	, 71	λ_2	λ_3	, Yı	λ_2	λ_3
U	0.3	0.3	38.892	53.674	78.807	40.928	51.869	66.455	66.768	79.049	93.454
C	0.3	0.5	50.032	75-462	108.321	50-470	71-419	95-913	78-793	$103 \cdot 133$	129.598
U	0.5	0.3	50.032	75-462	108.321	52.070	71.512	93·342	83·818	105.549	128.161
	0.5	0.5	60.435	96.573	130.658	60.810	89-935	120.402	94·703	127-745	$161 \cdot 188$
C	0.3	0.3	35.306	49-317	74.480	38-055	48.556	63.214	62.402	74.569	88.537
U	0.3	0.5	43.528	68.029	100.387	44·834	65.286	89.507	70.201	94.646	120.332
S	0.5	0.3	43·528	68.030	100.382	46.747	65.684	86.923	75-889	97.702	120.019
	0.5	0.5	51.515	86.302	119.026	53.136	81.466	111.039	83·241	116.165	148.177
U	0.3	0.3	25.706	39-029	56.711	28.199	40.620	51.939	39.870	63-805	75.475
U	0.3	0.5	28.137	49.723	74·764	29.750	49.529	69.750	41·252	$72 \cdot 120$	92-994
ГĻ	0.5	0.3	28.137	49.723	74.763	31.204	50-997	69·233	44.826	77-334	95.819
	0.5	0.5	30.540	60.139	79-884	32·746	59.162	82.471	46.312	84.292	109.162
U	0.3	0.3	32.078	46.211	68-976	31.558	42.485	57-933	47-305	59.739	76.312
S	0.3	0.5	39.737	63.918	95.457	38·486	58.251	82.673	57-234	79-941	106.793
U	0.5	0.3	42.577	66.161	94.690	42·243	60.930	81.925	64.094	85.666	107-528
	0.5	0.5	49.615	82.891	113.378	48.556	75.532	104.121	73.155	103-995	135-754
C	0.3	0.3	28.741	42·034	64.779	29.124	39-435	54.764	44.566	56.151	72.226
S	0.3	0.5	34.075	56.968	88·415	33-990	52.720	76.733	51.730	72.856	98.509
S	0.5	0.3	36.360	59-071	86.907	37-456	55.423	75-824	58.252	78.642	99.786
	0.5	0.5	41.581	73.360	102.650	42.055	67-783	95·331	64.852	93-754	124-248
C	0.3	0.3	16.898	31.566	47.796	19-819	31.824	43.618	31.863	46.018	57-647
S	0.3	0.5	17.781	39-027	58.052	20.961	38.602	57.564	33.606	53.555	73·349
Ĺ	0.5	0.3	18.795	41·224	59-295	22.677	41.747	58.788	37.095	59-477	77.663
	0.5	0.5	19.782	48·516	62.843	23.852	47.807	68·707	38.773	65-910	90-272
C	0.3	0.3	16.146	26.105	42.823	13.636	21.602	34.283	16.233	24.585	36.689
Ľ	0.3	0.5	17-959	35.525	53-422	15-939	30-925	$51 \cdot 108$	19-423	35.658	57-461
U	0.5	0.3	22.356	38·209	56-915	19-931	33.031	48-475	25.429	40.072	57-465
	0.5	0.5	22.850	44·363	65.766	20-990	39.076	61.379	27.126	47.058	70-990
										contin	ued overleaf

TABLE 1 First three frequency parameters for quadratic thickness variation (Figure 3(c)) TRIANGULAR PLATES WITH VARIABLE THICKNESS

	3/2	λ_3	34.625	51.893	52.569	63-952	24.423	33.416	36.634	41.888	77·024	110.327	100.885	132.528	72.456	101.631	93.664	121.027	57.606	70.780	68-354	606-62	62·160	88·869	84·144	110-309	57-840	80.551	77.196	99-492	ted overleaf
	$-0.5, \eta = $	λ_2	23.031	32·318	36.593	42.467	16.423	19.529	24.435	26.158	64.227	87-415	81.182	103.282	60.063	79-203	74-498	92.804	47.668	52.752	53.532	58-021	47.848	66.838	64.653	82.768	44.519	59-958	58.720	73.565	continu
	ي د	л, У,	15.315	17.797	23.201	24.611	7.789	7-941	$11 \cdot 134$	11.508	53.352	65·118	63.009	74.136	49.334	56.766	56.593	63-937	21.524	22.618	22.660	23.826	36.914	46.578	46.736	55-935	34.448	41.226	42.067	48.627	
	0.	λ_3	30-890	45.555	43.111	54.418	21.052	28.141	29.353	33.322	56.861	83-061	78.168	102.016	53.733	76.837	72.604	93·392	41.656	55.380	52.734	62.666	48.104	69-927	67-211	87-771	44.837	64-017	61.694	79.712	
inued	$= 0.0, \eta = 1$	λ_2	18.553	25-974	28.259	33.117	12.895	14.723	18.040	19-038	43.058	61-497	57-058	74.676	39-949	55.622	51.868	66.952	31.660	39-055	37-647	44.620	35.144	49.716	48.317	62·282	32.251	44·392	43.409	55.216	
BLE 1—cont	λ L	71	11.150	12.225	15.507	16.186	4.823	4.834	6.439	6.565	33.059	42·241	39.910	48.565	30.407	36.791	35.471	41.698	17.395	18-551	18.235	19-470	25.326	31.994	31.993	38-275	23.079	27.634	27-990	32.472	
TA	/3/2	la la	36.429	45.991	48.080	55.151	23.723	24.922	26.386	27·847	68-976	94.690	95.457	113-378	64·781	86.906	88·415	102.650	47.797	59-295	58.052	62.843	58.278	83.114	83.114	97-765	53.968	75-914	75-913	87.877	
	= $0.5, \eta = $	λ_2	20·887	27-825	30-747	35-356	13-437	14.292	17.859	18.360	46.211	66.161	63.918	82.891	42·034	59-071	56.968	73-360	31.566	41·224	39-027	48.516	39.588	55.610	55.610	71.196	35.587	49.008	49.008	62·330	
	л Ц	, 71	10.842	11.010	14.066	14.191	3.859	3.826	4.838	4.892	32.078	42.577	39.737	49.615	28-741	36.360	34.075	41.581	16.898	18.795	17.781	19.782	26.539	33.793	33.793	40.577	23.423	28.365	28.365	33·324	
		β	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	
		ъ	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	
	- c	1 ന	ပ	ĹĹ	S		U	ĹĹ	Ц		S	U	U		S	U	S		S	U	Ц		S	S	C		S	S	S		

B. SINGH AND S. M. HASSAN

$\gamma_3 - \gamma_3/2$ $\zeta = 0.0, \eta = 1.0$ $\zeta = -0.5, \eta = \sqrt{3}/2$ λ_3 λ_3 λ_4 λ_4 λ_4 λ_4 λ_3 λ_4 λ_4 λ_4 λ_3 λ_4 λ_4 λ_4 λ_3 λ_4		د		TAI	BLE 1-con	tinued		د		0.0
λ_2 λ_3 λ_1 λ_2 λ_3 λ_1 λ_2 λ_3 λ_2 λ_3 λ_1 λ_2 λ_3 λ_3 λ_4 λ_2 λ_3 25698 40.026 $11:513$ $24:396$ $33:895$ $17:617$ $33:098$ $42:863$ $32:401$ $44:885$ $12:2465$ $29:735$ $44:291$ $86:65$ $38:438$ $54:243$ $32:3505$ $47:671$ $12:2455$ $39:991$ $18:616$ $37:848$ $42:235$ $32:566$ $49:428$ $11:027$ $17:485$ $27:144$ $46:788$ $32:566$ $49:428$ $11:027$ $17:485$ $27:131$ $12:186$ $86:67$ $38:412$ $55:532$ $15:943$ $31:861$ $50:771$ $21:8494$ $45:781$ $32:566$ $49:428$ $14:474$ $25:630$ $39:36701$ $14:197$ $21:731$ $21:731$ $22:138$ $49:495$ $11:620$ $21:4748$ $20:6107$ $31:080$ $46:788$ $22:438$ $41:137$ $10:773$ $21:331$ $44:466$ $16:370$ $31:647$ $25:438$ $41:137$ $10:773$ $21:331$ $34:545$ $14:317$ $24:295$ $56:418$ $25:440$ $19:830$ $14:746$ $16:370$ $31:949$ $36:478$ $15:732$ $21:331$ $21:331$ $34:545$ $31:645$ $36:701$ $25:440$ $19:830$ $14:716$ $25:637$ $31:645$ $36:705$ $15:733$ $21:733$ $21:733$ $21:736$ $21:736$ $25:440$ </th <th></th> <th>ν.</th> <th>$= 0.5, \eta = \sqrt{2}$</th> <th>3/2</th> <th></th> <th>$\zeta = 0.0, \eta = 1$</th> <th>,</th> <th>" بر ا</th> <th>$= -0.5, \eta = 1$</th> <th>/3/2</th>		ν.	$= 0.5, \eta = \sqrt{2}$	3/2		$\zeta = 0.0, \eta = 1$,	" بر ا	$= -0.5, \eta = 1$	/3/2
25.698 40.026 11.513 24.396 33.895 17.617 33.098 42.805 32.401 44.885 12.2465 29.735 44.219 18.615 39.551 54.213 32.235 47.627 12.2465 39.4399 18.615 39.551 54.213 32.535 47.627 11.027 11.2455 35.935 51.7144 49.577 32.566 49.428 14.474 25.630 39.368 15.921 27.144 40.577 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 38.780 9833 14.716 23.9461 11.339 17.074 25.721 25.438 41.137 11.620 26.381 44.466 16.370 34.595 17.039 28.642 11.620 26.381 44.466 16.370 34.592 17.039 28.642 11.620 26.381 44.466 16.377 34.599 17.039 28.642 11.620 26.381 24.662 34.592 34.592 17.039 28.642 11.620 26.381 24.662 31.692 34.592 <th>γ'</th> <th></th> <th>λ_2</th> <th>λ_3</th> <th>γ,</th> <th>λ_{2}</th> <th>λ_3</th> <th>, 71</th> <th>λ_2</th> <th>λ_3</th>	γ'		λ_2	λ_3	γ,	λ_{2}	λ_3	, 71	λ_2	λ_3
32.401 44.885 12.405 29.735 44.219 18.766 38.438 54.683 32.201 41.885 12.275 30.099 41.349 18.615 39.551 54.213 $32.23.5$ 47.627 13.245 30.099 41.485 12.275 30.099 27.848 32.536 49.428 11.202 17.485 27.157 12.186 39.551 54.348 30.635 49.428 14.474 25.630 39.368 15.921 27.144 40.577 38.412 55.532 15.943 31.861 50.879 18.494 35.522 55.107 37.612 37.636 49.426 8.700 14.716 27.946 11.339 17.074 25.714 40.577 38.412 35.532 15.943 31.861 36.428 44.666 27.426 8.700 14.716 23.946 11.339 17.074 25.721 25.438 41.137 10.773 21.495 35.701 14.197 24.295 34.592 17.039 28.642 11.322 19.456 27.832 24.593 34.592 16.792 27.666 9.860 19.605 27.632 24.592 34.592 17.039 28.642 11.322 19.707 24.292 34.592 16.792 27.666 9.860 19.605 27.832 24.593 34.592 16.792 27.966 38.6701 12.926 27.848 26.714	9.82	2	25.698	40.026	11.513	24.396	33.895	17.617	33.098	42.805
32.401 44.885 12.275 30.099 43.490 18.615 39.551 54.213 39.235 47.627 13.245 35.085 51.134 19.824 44.387 64.344 22.137 35.076 11.027 17.485 27.157 12.186 18.609 27.848 30.635 49.428 13.4466 26.121 42.033 15.939 29.538 46.384 32.566 49.428 11.4716 23.946 11.339 17.074 25.721 38.412 55.723 15.943 31.861 35.704 11.703 24.295 36.428 23.681 38.780 9835 21.495 36.701 14.197 27.603 40.670 23.681 38.780 9835 21.495 36.701 14.197 26.203 40.670 23.681 38.780 9835 21.495 36.701 14.197 26.263 40.670 23.681 38.730 14.716 23.946 11.339 17.074 25.721 23.681 38.732 14.746 16.370 31.089 48.477 16.792 29.766 9.8832 14.746 16.370 31.632 23.642 11.620 25.643 32.525 36.428 16.792 27.946 19.831 12.028 27.724 25.643 38.731 14.766 27.332 22.945 16.776 92.3766 9.832 14.746 16.774 26.774 1	10.5(68	32.401	44.885	12.405	29.735	44·219	18.766	38.438	54.683
39-235 $47\cdot627$ $13\cdot245$ $35\cdot085$ $51\cdot134$ $19\cdot824$ $44\cdot387$ $64\cdot344$ 22:137 $35\cdot076$ $11\cdot027$ $17\cdot485$ $27\cdot157$ $12\cdot186$ $18\cdot609$ $27\cdot848$ $30\cdot635$ $45\cdot985$ $13\cdot486$ $26\cdot121$ $42\cdot083$ $15\cdot939$ $29\cdot528$ $46\cdot528$ $32\cdot566$ $49\cdot428$ $14\cdot474$ $25\cdot630$ $39\cdot368$ $15\cdot939$ $29\cdot528$ $46\cdot573$ $38\cdot412$ $55\cdot532$ $15\cdot943$ $31\cdot861$ $50\cdot879$ $18\cdot494$ $35\cdot222$ $55\cdot107$ $17\cdot506$ $29\cdot426$ $8\cdot700$ $14\cdot716$ $23\cdot946$ $11\cdot339$ $17\cdot074$ $25\cdot721$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot197$ $26\cdot030$ $40\cdot670$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot317$ $24\cdot295$ $36\cdot428$ $25\cdot9918$ $46\cdot045$ $11\cdot6202$ $21\cdot331$ $34\cdot545$ $14\cdot716$ $25\cdot721$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot716$ $25\cdot721$ $25\cdot438$ $11\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot767$ $25\cdot721$ $16\cdot792$ $27\cdot666$ $98\cdot802$ $17\cdot748$ $26\cdot714$ $21\cdot692$ $31\cdot843$ $16\cdot703$ $28\cdot642$ $11\cdot322$ $19\cdot655$ $21\cdot692$ $21\cdot692$ $31\cdot843$ $16\cdot703$ $28\cdot642$ $11\cdot322$ $19\cdot655$ $28\cdot489$ $41\cdot667$ $25\cdot477$ $25\cdot438$ $34\cdot546$ $11\cdot322$ $19\cdot65$ $28\cdot489$ $41\cdot667$ 25	10.5(88	32.401	44·885	12.275	30.099	43-499	18.615	39-551	54.213
22:137 35.076 11.027 17.485 27.157 12.186 18.609 27.848 30.635 45.985 13.486 26.121 42.083 15.939 29.528 46.288 32.566 49.428 14.474 25.630 39.368 15.921 27.144 40.577 38.412 55.532 15.943 31.861 50.879 18.494 35.222 55.107 17.506 29.426 8.700 14.716 23.946 11.339 17.074 25.721 25.438 41.137 10.773 21.331 34.545 14.317 24.295 56.428 25.943 38.780 9.8832 14.748 20.305 16.792 24.426 46.457 15.440 19839 8832 14.748 20.305 15.820 24.169 16.703 27.666 9860 27.687 14.317 22.945 34.542 16.703 28.642 11.322 19.605 25.933 13.026 21.692 31.843 17.039 28.642 11.322 19.605 27.687 21.692 31.843 17.039 28.642 11.322 19.653 22.657 31.692 27.736 17.039 28.642 11.322 19.663 22.736 21.692 31.843 16.703 28.642 11.322 19.683 21.687 21.692 31.843 17.039 28.642 11.322 19.465 25.933 12.677 23.736	11-41	6	39.235	47-627	13.245	35-085	51.134	19.824	44.387	64·344
30.635 45.985 13.486 26.121 42.083 15.939 29.528 46.288 32.566 49.428 14.474 25.630 39.368 15.921 27.144 40.577 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 38.412 55.532 15.943 31.861 50.879 18.494 35.522 55.107 23.681 38.780 9.835 21.495 36.701 14.197 25.721 40.670 25.438 41.137 10.773 21.331 34.545 11.4197 24.295 36.428 29.918 46.045 11.620 26.331 44.466 16.370 31.089 48.477 15.740 19.839 8.832 14.748 20.302 15.820 24.169 16.792 27.066 9.860 19.605 26.117 12.323 22.945 34.599 16.792 27.066 9.860 19.605 21.736 21.736 34.252 38.209 56.915 20.047 23.1687 14.488 26.714 40.217 38.209 56.915 20.047 23.153 21.736 27.373 34.252 38.209 56.915 20.047 23.1687 21.736 27.373 34.252 38.209 56.915 20.946 21.736 27.373 34.252 38.209 56.916 21.736 27.349 21.677 27.373 24.652 <td>13.74</td> <td>6</td> <td>22.137</td> <td>35.076</td> <td>11.027</td> <td>17-485</td> <td>27.157</td> <td>12.186</td> <td>18.609</td> <td>27.848</td>	13.74	6	22.137	35.076	11.027	17-485	27.157	12.186	18.609	27.848
32.566 49.428 14.474 25.630 39.368 15.921 27.144 40.577 38.412 55.532 15.943 31.861 50.879 18.494 35.222 55.107 38.412 55.532 15.943 31.861 50.879 18.494 35.252 55.107 23.681 38.780 9.835 21.495 36.701 14.197 26.030 40.670 25.438 41.137 10.773 21.331 34.545 14.317 24.295 36.428 29.918 46.045 11.620 26.381 44.466 16.370 31.089 48.477 15.440 19.839 8.832 14.748 20.302 10.339 15.820 24.169 16.792 27.066 9.860 19.605 26.117 12.323 22.945 34.599 17.039 28.642 11.322 19.456 25.953 13.026 21.692 31.843 17.039 28.642 11.322 19.456 25.953 $13.22.945$ 34.599 17.736 27.945 27.945 27.945 27.732 27.945 38.209 56.915 20.947 27.945 27.945 27.732 38.209 56.915 20.947 27.945 27.732 27.945 38.735 55.915 19.5923 29.668 21.692 21.736 38.735 55.915 19.732 27.946 12.731 27.945 38.735 56.916 $9.42.$	15.58	9	30.635	45.985	13.486	26.121	42·083	15.939	29.528	46.288
$38\cdot412$ $55\cdot532$ $15\cdot943$ $31\cdot861$ $50\cdot879$ $18\cdot494$ $35\cdot252$ $55\cdot107$ $17\cdot506$ $29\cdot426$ $8\cdot700$ $14\cdot716$ $23\cdot946$ $11\cdot339$ $17\cdot074$ $25\cdot721$ $23\cdot681$ $38\cdot780$ $9\cdot835$ $21\cdot495$ $36\cdot701$ $14\cdot197$ $26\cdot030$ $40\cdot670$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot317$ $26\cdot030$ $40\cdot670$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $34\cdot545$ $14\cdot317$ $24\cdot295$ $36\cdot428$ $25\cdot438$ $41\cdot137$ $10\cdot773$ $21\cdot331$ $24\cdot545$ $14\cdot317$ $24\cdot295$ $36\cdot428$ $27\cdot400$ $19\cdot839$ $8\cdot832$ $14\cdot748$ $20\cdot302$ $10\cdot339$ $15\cdot820$ $24\cdot169$ $17\cdot039$ $28\cdot642$ $11\cdot322$ $19\cdot665$ $26\cdot117$ $12\cdot323$ $22\cdot945$ $31\cdot692$ $17\cdot039$ $28\cdot642$ $11\cdot322$ $19\cdot456$ $25\cdot953$ $13\cdot266$ $24\cdot796$ $31\cdot843$ $17\cdot039$ $28\cdot642$ $11\cdot322$ $19\cdot456$ $25\cdot953$ $13\cdot266$ $24\cdot796$ $17\cdot039$ $28\cdot642$ $11\cdot322$ $19\cdot456$ $25\cdot953$ $13\cdot266$ $24\cdot796$ $17\cdot039$ $28\cdot642$ $11\cdot322$ $19\cdot692$ $21\cdot736$ $27\cdot736$ $24\cdot752$ $38\cdot209$ $56\cdot915$ $20\cdot047$ $32\cdot155$ $45\cdot682$ $28\cdot489$ $41\cdot861$ $55\cdot477$ $38\cdot209$ $56\cdot915$ $20\cdot047$ $32\cdot155$ $45\cdot682$ $28\cdot489$ $41\cdot861$ $57\cdot472$ $38\cdot209$ $56\cdot915$ $20\cdot047$ 3	17-94	5	32.566	49.428	14.474	25.630	39.368	15.921	27.144	40.577
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	18.654		38-412	55-532	15.943	31.861	50.879	18.494	35.252	55.107
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	8.862	•	17.506	29-426	8.700	14.716	23.946	11.339	17-074	25.721
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	9.010	_	23.681	38.780	9.835	21.495	36.701	14.197	26.030	40.670
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	10.680	_	25.438	$41 \cdot 137$	10.773	21.331	34.545	14.317	24.295	36.428
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	10.791		29.918	46.045	11.620	26.381	44-466	16.370	31.089	48.477
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	10.630	_	15.440	19.839	8.832	14.748	20.302	10.339	15.820	24.169
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	10.914		16.792	27.066	9.860	19.605	26.117	12.323	22.945	34.599
18.568 34.371 12.023 22.627 31.687 14.488 26.714 40.217 $26\cdot105$ $42\cdot823$ 14.574 $20\cdot461$ $29\cdot699$ $21\cdot736$ $27\cdot373$ $34\cdot252$ $38\cdot209$ $56\cdot915$ $20\cdot047$ $32\cdot155$ $45\cdot682$ $28\cdot489$ $41\cdot861$ $55\cdot477$ $35\cdot525$ $55\cdot915$ $20\cdot047$ $32\cdot155$ $45\cdot682$ $28\cdot489$ $41\cdot861$ $55\cdot477$ $35\cdot525$ $55\cdot766$ $19\cdot591$ $34\cdot355$ $50\cdot924$ $27\cdot086$ $42\cdot172$ $58\cdot004$ $20\cdot887$ $36\cdot428$ $9\cdot875$ $16\cdot774$ $24\cdot866$ $13\cdot331$ $23\cdot011$ $28\cdot965$ $30\cdot747$ $48\cdot080$ $12\cdot044$ $25\cdot435$ $37\cdot979$ $15\cdot092$ $31\cdot869$ $44\cdot411$ $27\cdot825$ $45\cdot991$ $9\cdot432$ $19\cdot482$ $32\cdot173$ $12\cdot677$ $23\cdot078$ $32\cdot726$ $30\cdot747$ $48\cdot080$ $12\cdot044$ $25\cdot435$ $37\cdot979$ $15\cdot092$ $31\cdot915$ $46\cdot025$ $30\cdot747$ $48\cdot080$ $12\cdot044$ $25\cdot435$ $37\cdot979$ $12\cdot677$ $23\cdot078$ $32\cdot726$ $31\cdot735$ $55\cdot151$ $11\cdot762$ $26\cdot879$ $41\cdot851$ $14\cdot673$ $31\cdot915$ $46\cdot025$ $13\cdot437$ $23\cdot723$ $3\cdot582$ $11\cdot787$ $18\cdot033$ $3\cdot810$ $14\cdot857$ $22\cdot722$ $17\cdot859$ $26\cdot386$ $4\cdot146$ $12\cdot677$ $23\cdot078$ $32\cdot726$ $17\cdot859$ $26\cdot386$ $4\cdot1211$ $16\cdot673$ $31\cdot915$ $46\cdot025$ $17\cdot859$ $26\cdot386$ $4\cdot1213$ $11\cdot608$ $21\cdot$	13.538		17.039	28.642	11.322	19.456	25.953	13.026	21.692	31.843
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	13·647		18.568	34.371	12.023	22.627	31.687	14.488	26.714	40.217
38.209 56.915 20.047 32.155 45.682 28.489 41.861 55.477 35.525 53.422 14.853 25.063 39.233 20.678 29.165 40.792 44.363 65.766 19.591 34.355 50.924 27.086 42.172 58.004 20.887 36.428 9.875 16.774 24.866 13.331 23.011 28.965 20.887 36.428 9.875 16.774 24.866 13.331 23.011 28.965 30.747 48.080 12.044 25.435 37.979 15.092 31.869 44.411 27.825 45.991 9.432 19.482 32.173 12.677 23.078 32.726 35.356 55.151 11.762 26.879 41.851 14.673 31.915 46.025 13.437 23.723 3.582 11.787 18.033 3.810 14.857 22.722 17.859 26.386 4.146 14.812 25.134 4.211 16.655 28.467 17.859 26.386 4.146 14.812 25.134 4.211 16.655 28.467 14.292 27.847 4.213 14.781 25.148 3.924 14.468 22.945 14.292 27.847 4.213 14.781 26.140 4.353 16.677 28.624	16.146		26.105	42.823	14.574	20.461	29-699	21.736	27.373	34.252
$35\cdot525$ $53\cdot422$ $14\cdot853$ $25\cdot063$ $39\cdot233$ $20\cdot678$ $29\cdot165$ $40\cdot792$ $44\cdot363$ $65\cdot766$ $19\cdot591$ $34\cdot355$ $50\cdot924$ $27\cdot086$ $42\cdot172$ $58\cdot004$ $20\cdot887$ $36\cdot428$ $9\cdot875$ $16\cdot774$ $24\cdot866$ $13\cdot331$ $23\cdot011$ $28\cdot965$ $30\cdot747$ $48\cdot080$ $12\cdot044$ $25\cdot435$ $37\cdot979$ $15\cdot092$ $31\cdot869$ $44\cdot411$ $27\cdot825$ $45\cdot991$ $9\cdot432$ $19\cdot482$ $32\cdot173$ $12\cdot677$ $23\cdot078$ $32\cdot726$ $35\cdot356$ $55\cdot151$ $11\cdot762$ $26\cdot879$ $41\cdot851$ $14\cdot673$ $31\cdot915$ $46\cdot025$ $13\cdot437$ $23\cdot723$ $3\cdot582$ $11\cdot787$ $18\cdot033$ $3\cdot810$ $14\cdot857$ $22\cdot722$ $17\cdot859$ $26\cdot394$ $41\cdot812$ $25\cdot134$ $4\cdot211$ $16\cdot655$ $28\cdot467$ $17\cdot859$ $26\cdot391$ $11\cdot608$ $21\cdot489$ $3\cdot924$ $14\cdot468$ $22\cdot945$ $14\cdot292$ $27\cdot847$ $4\cdot213$ $14\cdot781$ $26\cdot140$ $4\cdot353$ $16\cdot677$ $28\cdot624$	22.356		38·209	56.915	20.047	32.155	45.682	28.489	41.861	55-477
44.363 65.766 19.591 34.355 50.924 27.086 42.172 58.004 20.887 36.428 9.875 16.774 24.866 13.331 23.001 28.965 30.747 48.080 12.044 25.435 37.979 15.092 31.869 44.411 27.825 45.991 9.432 19.482 32.173 12.677 23.078 32.726 35.356 55.151 11.762 26.879 41.851 14.673 31.915 46.025 13.437 23.723 3.582 11.787 18.033 3.810 14.857 22.722 17.859 26.386 4.146 14.812 25.134 4.211 16.655 28.467 14.292 27.847 4.213 11.608 21.489 3.924 14.468 22.945 18.360 27.847 4.213 14.781 26.140 4.353 16.677 28.624	17-959		35.525	53.422	14.853	25.063	39-233	20.678	29.165	40.792
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	22.850		44·363	65·766	19-591	34.355	50-924	27·086	42.172	58·004
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10.842		20.887	36.428	9.875	16.774	24·866	13.331	23.001	28.965
27.825 45.991 9.432 19.482 32.173 12.677 23.078 32.726 35.356 55.151 11.762 26.879 41.851 14.673 31.915 46.025 13.437 23.723 3.582 11.787 18.033 3.810 14.857 22.722 17.859 26.366 4.146 14.812 25.134 4.211 16.655 28.467 17.859 26.3591 11.608 21.489 3.924 14.468 22.722 18.360 27.847 4.213 14.781 26.140 4.353 16.677 28.645	14.066		30.747	48.080	12.044	25.435	37-979	15.092	31.869	44-411
35·356 55·151 11·762 26·879 41·851 14·673 31·915 46·025 13·437 23·723 3·582 11·787 18·033 3·810 14·857 22·722 17·859 26·366 4·146 14·812 25·134 4·211 16·655 28·467 14·292 3·591 11·608 21·489 3·924 14·468 22·945 18·360 27·847 4·213 14·781 26·140 4·353 16·677 28·624	11.010		27.825	45.991	9.432	19.482	32.173	12.677	23.078	32.726
13·437 23·723 3·582 11·787 18·033 3·810 14·857 22·722 17·859 26·386 4·146 14·812 25·134 4·211 16·655 28·467 17·859 26·386 4·146 14·812 25·134 4·211 16·655 28·467 14·292 3·591 11·608 21·489 3·924 14·468 22·945 18·360 27·847 4·213 14·781 26·140 4·353 16·677 28·624	14.191		35-356	55.151	11.762	26.879	41·851	14.673	31.915	46.025
17.859 26.386 4.146 14.812 25.134 4.211 16.655 28.467 14.292 24.922 3.591 11.608 21.489 3.924 14.468 22.945 18.360 27.847 4.213 14.781 26.140 4.353 16.677 28.624	3-85	6	13.437	23.723	3.582	11.787	18.033	3.810	14.857	22.722
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	4.83	~	17-859	26.386	4.146	14.812	25.134	4.211	16.655	28-467
18·360 27·847 4·213 14·781 26·140 4·353 16·677 28·624	3.82	9	14·292	24-922	3.591	11.608	21-489	3.924	14.468	22.945
	4.89	2	18.360	27·847	4.213	14.781	26.140	4.353	16.677	28.624

TRIANGULAR PLATES WITH VARIABLE THICKNESS

TABLE 1—continued

	3/2	λ_3	25.425	41.519	31.716	43·741	20.193	31.530	24.461	33.172	20.466	28.477	23.585	29.284	15.112	23.582	18.972	24.876	25.158	17.703	25.119	21.006	27.649	17-537	23.022	20.118	25.313
	$-0.5, \eta = $	λ_2	18.602	29.365	$21 \cdot 777$	29.791	14.569	20.522	15.929	20.861	14.612	20.137	16.062	20.540	660.6	14.388	11.504	15.190	15-337	10.631	16.115	13.239	17.304	11.478	15.147	13.588	16.548
	<i>с</i> " =	y,	13.322	17-385	13.430	16.575	8.828	9-407	8.036	8·701	8.674	9.385	8.568	9-432	5.443	6.475	5.816	6.500	6.5534	6·203	8.708	7.263	9.105	6.523	9.343	7.780	9-949
	0.	λ3	24.119	37.682	32.554	42.618	19.806	30.449	26.230	34.179	17.790	22·034	19.180	23.954	16.450	23.053	20.663	24.313	24.580	16.883	22.850	20.235	25.402	17.545	23.581	21.532	27.108
inued	$= 0.0, \eta = 1.$	λ_2	16.483	25.532	21.010	27.618	13.062	19-224	15.894	20.720	13.940	18.631	16.738	19-320	10.349	15.523	13.678	16.971	16-902	12.553	17-450	15.887	18.597	14.346	18-235	17.871	20.111
3LE 1—conti	ν v	Y'	11.127	14.589	11.833	14.383	7.300	8·313	6.928	7.956	8-919	10.493	8-944	10.565	6.253	7-215	6.770	7.286	7·2571	7.390	10.328	9.298	11.290	7·844	11.554	10.218	12.962
TAF	3/2	λ3	35-076	49-428	45.985	55-532	29.426	$41 \cdot 137$	38.780	46.045	19-839	28.642	27.066	34·371	21.722	26.098	26.098	28.341		19-899	29-777	29-777	37.107	21.815	25.321	25.321	26.802
	= $0.5, \eta = \sqrt{3}$	λ_2	22.137	32.566	30.635	38-412	17.506	25.438	23.681	29-918	15.440	17.039	16.792	18.568	15.198	21.554	21.554	25-427		17.266	18.645	18.645	19.406	21.072	23.375	23.375	26.100
	ي بر	y,	13.749	17-945	15.586	18.654	8.862	10.680	9.010	10.791	10.630	13.538	10.914	13.647	9.057	10.126	10.126	10.529		11.031	15.188	15.188	18.093	11.748	16.765	16.765	20-465
		β	0·3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5	nce [1]	0.3	0.5	0.3	0.5	0.3	0.5	0.3	0.5
		ъ	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5	Refere	0.3	0.3	0.5	0.5	0.3	0.3	0.5	0.5
	- c	1 M	ц	S	U		Ц	S	S		ц	S	Ц		Ц	Ц	C			Ц	ĹĹ	S		Ц	ĹĹ	Ц	

A comparison of the results of references [1] and [2] can be found in reference [2]. Calculations for various values of α' and β' have been carried out. The results agree completely with reference [2] in all the cases at N = 28. So to avoid duplication of results, this comparison has been omitted here. In Table 1, results are given for quadratic variation. Some of these reduce to linear variation in special cases.

3.3.3. Quadratic thickness variation

Here, M = 6. So six parameters are at our disposal. The following special quadratic variations have been examined in detail:

(1)
$$f = (4\beta - 1)L_1 + (4\alpha - 1)L_2 + (2 - 4\beta)L_1^2 + 4(1 - \alpha - \beta)L_1 L_2 + (2 - 4\alpha)L_2^2$$

(Figure 3(c)). (15)

The results for this case are given in Table 1. If $\alpha = \beta = 0.5$, this reduces to linear variation given above with $\alpha' = 1$ and $\beta' = 0$.

(2)
$$f = 1 - 4(1 - \beta)L_1 - 4(1 - \alpha)L_2 + 4(1 - \beta)L_1^2 + 4(1 - \alpha - \beta + \gamma)L_1L_2 + 4(1 - \alpha)L_2^2$$

(Figure 3(d)). (16)

The results are given in Table 2. If $\alpha = \beta = \gamma = 1$, this reduces to uniform thickness. These choices have been made by assigning appropriate values to *f* at vertices and mid-points of sides. Comparison has also been made with references [3–5, 7–16] for the case of uniform thickness.

3.3.4. Cubic thickness variation

In this case M = 10 and so one has the freedom of choosing ten parameters. Numerical results for the following special case have been obtained

$$f = 1 + 27(\alpha - 1)L_1 L_2 (1 - L_1 - L_2) = 1 + 27(\alpha - 1)L_1 L_2 L_3 \quad \text{(Figure 3(e))}. \quad (17)$$

The results are given in Table 3. This corresponds to f = 1 on the boundary and $f = \alpha$ at the centroid of the triangle. Not that $\alpha = 1$ corresponds to uniform thickness. In Table 3 results are given for $\alpha = 0.5$, 1.5, for the three types of triangle and all combinations of boundary conditions. The results for uniform thickness are not given because these are the same as in Table 2.

A large number of approximations (up to N = 36) have been worked out to ensure convergence of results up to at least three significant figures in all cases. This is clearly illustrated in Table 4 in which results for fundamental frequency parameters are reported for various values of N for the set of parameters specified in the table. An interesting special case arises for the isosceles right-angled triangle of uniform thickness when all sides are simply-supported. As reported by Gorman [8], the exact values of the first three frequency parameters in this case are $5\pi^2$, $10\pi^2$ and $13\pi^2$ giving 49.348, 98.696 and 128.305, respectively. The corresponding values obtained here for N = 21, 28, 35 and 36 are as follows:

N	λ_1	λ_2	λ_3	
21	49.359	98.948	130.296	
28	49.348	98.833	128.624	
35	49.348	98.832	128.433	
36	49.348	98.700	128.433	

It is clear that the first frequency parameter has converged to all the five significant figures—the second differs from the exact value only in the last figure by 4. So it can be

ound, * for	/3/2	λ_3	174.550	174.550	178.484	134.550	272.217						155.389	89.483	161.518	121.264	243.064				103.917	66.744	109.105	82·246	155.072			159.644	155.399	161-518	ued overleaf
for lower b	$-0.5, \eta = \sqrt{2}$	λ_2	129-477	129-477	138.526	100.242	207·830						113-513	64.029	123.219	88·514	183.025				76.664	52-407	83·501	61.799	$111 \cdot 128$			113.762	113.513	123-219	contin
· bound, L	ي بر	71	89.713	89.713	100.814	72·814	140.172						75-947	50.685	85-488	60.630	118.670				41.333	31.558	47-461	34.630	58-484			$74 \cdot 107$	75-947	85-488	
U for upper	0.	λ_3	124.938	124-938	125-751	93.672	194.822		194.8	194.8			113.634	62.390	113-092	84.531	173.738		173.7	176.7	79.065	49.230	75·294	59.406	109.028		109.0	$111 \cdot 133$	113.634	113-092	
ure 3(d)) (1	$= 0.0, \eta = 1$	λ_2	97.150	97.150	100.326	72-436	157-789		157-8 157-8	1.1.61	D	Γ	84.245	42.350	89.371	64.121	138.848		138.8	138-9	55.667	33.655	60.857	44·838	86.698		86.7	86.393	84·245	89.371	
riation (Fig = 0·333)	νς.	y1	59.123	59.123	62.902	45.878	93.790	93·800	93·79	93.80	94.155	93·404	49·182	29.712	54.022	38-969	78.893	78.89	78.89	78.91	29.479	22·099	32.678	25·009	$41 \cdot 111$	41.12	41.11	50.169	49·182	54.022	
uickness va v	3/2	λ_3	119-717	119-717	119-717	87-027	189.007	189.22			D	Γ	106.681	53.652	105.225	77-915	165.319	165.52			72·042	44.178	67.048	$53 \cdot 809$	101.791	101.85		105-225	106.681	105-225	
quadratic tl	= $0.5, \eta = $	λ_2	117.079	117.079	117.079	87.027	189.007	189.05			189.116	188.892	98.870	47.216	103.483	74.955	164.989	165.12			60.529	33·224	65.554	48.403	95.827	95.891		103.483	98.870	103.483	
neters for	د =	γ1	62.674	62·674	62.674	46.972	99-020	99-022			99.044	98-989	49.831	26.359	52.847	38.928	81.601	81.604			27.576	19.007	29-928	23.610	40.016	40.022		52.847	49.831	52.847	
) paran		λ	1.0	0.5	0.5	0.5	$1 \cdot 0$	[3]	10]	<u>c</u>	15]	15]	$1 \cdot 0$	0.5	0.5	0.5	$1 \cdot 0$	[3]	10]	[5]	$1 \cdot 0$	0.5	0.5	0.5	$1 \cdot 0$	[3]	10]	$1 \cdot 0$	0.5	0.5	
(suanba		β	0.5	$1 \cdot 0$	0.5	0.5	$1 \cdot 0$	ference	erence [terence	erence [erence [0.5	0.1	0.5	0.5	$1 \cdot 0$	ference	erence [[erence	0.5	0.1	0.5	0.5	$1 \cdot 0$	ference	erence [0.5	$1 \cdot 0$	0.5	
hree fr		8	0.5	0.5	$1 \cdot 0$	0.5	$1 \cdot 0$	Rei	Ref	Ke	Ref	Ref	0.5	0.5	$1 \cdot 0$	0.5	$1 \cdot 0$	Rei	Ref	Rei	0.5	0.5	$1 \cdot 0$	0.5	$1 \cdot 0$	Rei	Ref	0.5	0.5	$1 \cdot 0$	
First t	- c	1 M	С	U	U								U	U	S						U	U	Ľ					U	S	C	

4 .* 4 111 $\overline{\zeta}$ 2 E L TABLE 2 1.:.1 ÷ f.

40

B. SINGH AND S. M. HASSAN

$ \begin{array}{c} & \eta = \sqrt{3/2} \\ \hline & \lambda = \sqrt{3/2} \\ \hline & 2 \\ \hline \hline & 2 \\ \hline & 2 \\ \hline & 2 \\ \hline \hline \hline \hline & 2 \\ \hline \hline \hline \hline & 2 \\ \hline \hline \hline \hline \hline \hline \hline \\ \hline \hline \hline \hline \hline \hline \hline \hline \hline \hline$	$ \frac{\xi}{2} = 0.5, \ \eta = \sqrt{3/2} $ $ \frac{\xi}{28} \qquad 74.955 \qquad 1 \\ 0.1 \qquad 164.989 \qquad 1 \\ 0.1 \qquad 165.12 \qquad 1 \\ 0.1 \qquad 164.989 \qquad 1 \\ 0.1 \qquad 165.12 \qquad 1 \\ 0.2 \qquad 33 \qquad 86.490 \\ 142.96 \qquad 1 \\ 142.96 \qquad 1 \\ 142.96 \qquad 1 \\ 142.96 \qquad 1 \\ 15.314 \\ 65 \qquad 75.36 \\ 65 \qquad 75.36 \\ 65 \qquad 554 \\ 10 \qquad 95.827 \qquad 1 \\ 10 \qquad 95.827 \qquad 1 \\ 11 \qquad 295.891 \qquad 1 \\ 17 \qquad 48.758 \\ 10 \qquad 48.758 \\ $	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{split} \vec{\xi} = 0.5, \ \eta = \sqrt{3/2} \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_1 \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_1 \qquad \lambda_4 \qquad$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$ \begin{array}{c} n \\ n $	$ \frac{\xi}{\xi} = 0.5, \ \eta = \sqrt{3/2} $ $ \frac{\xi}{28} = 0.5, \ \eta = \sqrt{3/2} $ $ \frac{1}{28} \qquad \lambda_2 \qquad \lambda_3 \qquad \lambda_4 \qquad \lambda_5 \qquad \lambda_4 \qquad \lambda_5 \qquad \lambda_6 \qquad$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

TABLE 2—continued

TRIANGULAR PLATES WITH VARIABLE THICKNESS

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				٢		TABLE	2-contin	ued ^		3	L C	
β γ λ_1 λ_2 λ_3 λ_4 λ_5 λ	β γ λ_1 λ_2 λ_1 λ_2 λ_3 λ_4 λ_2 λ_3 λ_4 λ_3 λ_4 λ_3 λ_4 λ_3 λ_4 λ				۳ ۳ ۲	$=0.5, \eta = $	3/2	μ J L	$= 0.0, \eta = 1$	ọ.	ا پن	$-0.5, \eta = $	(3/2
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0.5 0.5 10-275 26-800 32-905 10-866 37-315 15-842 32-052 53-76 10 10 10 26-561 75-314 84-350 31-78 75-131 84-446 75-131 teftrance 31 26-565 75-364 84-300 31-78 77-313 84-446 75-131 84-446 75-131 34-366 57-103 36-565 75-312 48-446 75-131 31-240 57-030 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 55-307 75-644 75-645 53-933 25-933 25-3077 75-645 57-964 57-963 36-56 57-964 57-964 57-963 36-56 57-964 57-963 36-56 57-964 57-963 36-56 57-964 57-964 57-964 57-964 57-964 57-964 57-964 57-964 57-964		β	λ	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	05 05 13707 38.549 41-503 16-566 37-524 49-006 27-312 48.446 73-13 10 26-561 75-314 84-330 31-78 72-801 94-375 51-280 95-310 136-360 11 26-561 75-314 84-330 31-78 72-801 94-375 51-240 75-313 48-430 53-067 53-300 53-310 136-360 53-300 53-300 53-300 53-300 53-300 53-300 53-300 53-300 53-300 53-300 53-300 55-340 75-300 53-300		0.5	0.5	10.275	26.800	32.905	10.887	26.152	35-315	15.842	32.052	53.767
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.5	0.5	13.707	38.549	41.503	16.566	37-524	49.006	27.312	48-446	73.132
	tefference [3] 26-565 75-36 84-432 31-78 7-81 94-36 105 10 5-247 23-171 25-440 7-065 23-389 34-025 12-118 31-240 53-009 10 0-5 0-5 5-247 23-171 25-440 7-065 23-389 34-025 12-118 31-240 53-009 0-5 0-5 5-3 3822 17-776 17-943 5-212 18-074 24-587 8885 24-575 42-015 0-5 0-5 3-5 35-015 38-401 12-645 35-935 52-933 23-077 50-277 79-644 0-5 0-5 3-5-35 35-012 38-902 35-012 56-032 23-470 77-644 0-5 0-5 3-841 12-645 35-944 5-033 36-575 79-644 0-5 0-5 38-922 35-132 38-754 10-202 146-765 10 1-0 89-922 35-132 38-964 91-915 31-925 10-4202 146-765 10 <td< td=""><td></td><td>$1 \cdot 0$</td><td>$1 \cdot 0$</td><td>26.561</td><td>75.314</td><td>84.350</td><td>31.783</td><td>72·801</td><td>94.375</td><td>51.280</td><td>95.310</td><td>136.363</td></td<>		$1 \cdot 0$	$1 \cdot 0$	26.561	75.314	84.350	31.783	72·801	94.375	51.280	95.310	136.363
efference [10] 31.78 72.81 94.36 0.5 1.0 5.247 23.171 25.440 7.065 23.389 34.025 12.118 31.240 53.009 0.5 0.5 6.032 23.171 25.440 7.065 23.389 34.025 12.40 53.005 0.5 0.5 6.032 23.172 25.440 7.065 23.389 34.025 12.40 53.007 0.5 0.5 38.71 17.943 5.212 18.074 24.575 42.015 1.0 1.0 8.922 35.155 38.503 12.645 35.935 22.933 23.077 79.614 kefrence [3] 8.922 35.155 38.503 12.645 35.935 24.575 42.015 0.5 1.0 1.0 8.922 35.153 88.575 104.202 146.765 0.5 1.06613 31.7561 10.6523 68.575	efference [10] 31.78 72.81 94.36 0.5 10 5.247 23.171 25.440 7.065 23.389 34.025 12.118 31.240 53.003 0.5 0.5 5.247 23.171 25.440 7.065 23.389 34.013 51.2118 31.240 53.003 0.5 0.5 5.347 23.171 25.440 7.065 23.385 34.013 51.018 31.240 53.003 0.5 0.5 5.337 23.471 25.440 7.065 23.385 34.015 57.903 34.613 50.277 79.644 57.903 10 1.0 8.921 35.015 38.403 12.645 35.913 24.575 140.702 146.765 10 1.0 1.0 8.922 38.503 12.645 35.933 68.575 104.202 146.765 10 1.0 52.847 103.483 105.225 47.061 81.764 106.523 68.575 104.202 146.765	Refe	rence [[3]	26.565	75.36	84-432	31.78					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	0-5 1-0 5-247 23-171 25-440 7-065 23-389 34-025 12-118 31-240 53-005 0-5 0-5 5-247 23-171 25-440 7-065 23-389 34-025 12-118 31-240 53-035 0-5 0-5 5-347 23-171 25-440 7-065 25-933 35-503 35-935 52-933 35-505 57-969 1-0 1-0 8-921 35-055 38-944 12-645 35-935 52-933 23-077 50-277 79-644 Reference [3] 8-922 38-155 38-505 12-64 12-645 35-935 52-933 23-077 50-277 79-644 Reference [9] 8-922 38-162 12-64 12-645 35-935 52-933 23-077 50-277 79-644 Reference [9] 8-922 38-162 12-64 12-645 35-93 52-93 23-077 50-277 79-64 Reference [9] 8-922 38-105 103-483	tefer	rence [10]				31.78	72.81	94.36			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		0.5	$1 \cdot 0$	5.247	23.171	25.440	7.065	23.389	34.025	12.118	31.240	53·009
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$1 \cdot 0$	0.5	5.247	23.171	25.440	7.065	23.389	34.025	12.118	31.240	53.009
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		0.5	0.5	6.032	24.429	24.470	8.548	25.495	34.613	15.603	36.636	57.969
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			0.5	0.5	3.872	17.776	17.943	5.212	18.074	24.587	8.885	24.575	42.015
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$ \begin{array}{llllllllllllllllllllllllllllllllllll$		1.0	1.0	8.921	35.095	38·484	12.645	35.935	52.933	23.077	50.277	79.644
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	$ \begin{array}{llllllllllllllllllllllllllllllllllll$	Refe	stence [[3]	8.922	35.155	38.503	12.64					
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Reference [10] 12-65 35.94 52.93 0.5 10 52.847 103.483 105.225 47.061 81.764 106.523 68.575 104.202 146.765 0.5 0.5 52.847 103.483 105.225 47.061 81.764 106.523 68.575 104.202 146.765 0.5 0.5 39.870 106.681 44.819 81.857 101.948 66.414 102.871 139.882 0.5 0.5 38.928 77.915 34.662 59.984 77.588 51.723 223.536 0.5 0.5 38.1604 165.12 165.52 73.39 131.6 165.1 105.223 146.765 Reference [10] 81.604 165.12 165.52 77.34 77.549 109.5263 165.723 223.536 Reference [10] 81.604 165.12 165.52 73.39 131.6 165.1 129.723 129.723 129.723 129.723 229.7277 89.671 129.711 129.711 129.711 129.711 <t< td=""><td>Refe</td><td>srence [</td><td>6</td><td>8.922</td><td>35.132</td><td>38.505</td><td></td><td></td><td></td><td></td><td></td><td></td></t<>	Refe	srence [6	8.922	35.132	38.505						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Sefer	rence [10]				12.65	35-94	52.93			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		0.5	$1 \cdot 0$	52.847	103.483	105.225	47·061	81.764	106.523	68.575	104.202	146.765
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		1.0	0.5	52.847	103.483	105.225	47.061	81.764	106.523	68.575	104.202	146.765
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		0.5	0.5	49.831	98.870	106.681	44·819	81.857	101.948	66.414	102.871	139.882
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.5	0.5	38-928	74.955	77-915	34.662	59-984	77-588	51.723	77-049	109.883
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Reference [3] 81·604 165·12 165·52 73·4 Reference [10] 73·39 131·7 165·0 Reference [11] 73·38 132·95 168·89 0·5 1·0 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 0·5 1·0 0·5 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 0·5 41·233 86·490 92·974 37·824 72·163 90·455 55·384 91·015 132·666 0·5 0·5 0·5 1/233 86·490 92·974 37·824 72·163 90·455 55·195 90·280 125·286 0·5 0·5 1·0 1·0 1/42·40 143·475 60·538 114·565 145·833 87·101 143·808 198·		$1 \cdot 0$	$1 \cdot 0$	81.601	164.989	165.319	73.395	131.582	165.049	105.263	165.723	223.536
teference [10] T3:39 131.6 165.1 T3:39 131.7 165.0 T3:38 132.95 168.89 T5:38 132.95 10.0 5 41:233 86.490 92:974 37.84 72.163 90:455 55:195 90:280 125:288 0:55 0:5 31:735 64.892 67.727 29:060 52:530 69:177 42:481 67:298 98:367 1.0 1.0 66:177 142:740 143:475 60:538 114:565 145:83 87:101 143:808 198:311 Reference [3] 66:18 142:96 143:73 60:54 114.6 145:85 145:89 198:311 Reference [10] terrece [10] terrece [10] terrece [3] 66:57 114.7 145:85 145:95 terrece [4] 60:57 114.7 145:85	teference [10] 73·39 131·6 165·1 Reference [5] 73·39 131·7 165·0 Reference [11] 73·38 132·95 168·89 0·5 1·0 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 1·0 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 0·5 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 0·5 41·233 86·490 92·974 37·824 72·163 90·455 55·195 90·280 125·286 0·5 0·5 0·5 1·0 142·740 143·475 60·538 114·565 145·833 87·101 143·808 198·311 Reference [10] 66·18 142·96 143·73 60·54 114·6 145·9 60·57 114·7 145·8 Reference [5] 60·54 114·7 145·9 <td>Refe</td> <td>rence [</td> <td>[3]</td> <td>81.604</td> <td>165.12</td> <td>165.52</td> <td>73-4</td> <td></td> <td></td> <td></td> <td></td> <td></td>	Refe	rence [[3]	81.604	165.12	165.52	73-4					
Reference [5] 73:39 131.7 165-0 Reference [11] 73:38 132.95 168.89 0.5 1.0 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1.0 0.5 1.0 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1.0 0.5 0.5 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1.0 0.5 0.5 41:233 86:490 92:974 37:824 72:163 90:455 55:195 90:280 125:288 0.5 0.5 0.5 31:735 64:490 92:974 37:824 72:163 90:455 55:195 90:280 125:288 1.0 1.0 1.0 66:177 142:740 143:475 60:538 114:565 145:833 87:101 143:808 198:311 Reference [10] 66:18 142:740 143:475 60:54 114:6 145:9	Reference [5] 73·39 131·7 165·0 Reference [11] 73·388 132·95 168·89 0·5 1·0 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 41·233 86·490 92·974 38·462 69·913 96·232 57·277 89·671 129·711 1·0 0·5 0·5 41·233 86·490 92·974 37·824 72·163 90·455 55·384 91·015 132·666 0·5 0·5 0·5 1·0 1/233 86·490 92·974 37·824 72·163 90·455 55·384 91·015 132·666 0·5 0·5 0·5 1·10 1·0 1·0 1·0 60·593 114·565 145·833 87·101 143·808 198·311 Reference [10] 66·18 142·96 143·77 22·060 52·530 69·177 42·	Sefer	rence [10]				73.39	131.6	165.1			
Reference [11] 73:388 132:95 168:89 0:5 1:0 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1:0 0:5 1:0 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1:0 0:5 0:5 41:233 86:490 92:974 38:462 69:913 96:232 57:277 89:671 129:711 1:0 0:5 0:5 41:233 86:490 92:974 37:824 72:163 90:455 55:195 90:280 125:588 0:5 0:5 31:735 64:490 92:974 37:824 72:163 90:455 55:195 90:280 125:588 1:0 1:0 1:0 66:177 142:740 143:475 60:538 114:565 145:833 87:101 143:808 198:311 Reference [3] 66:18 142:740 143:475 60:54 114:65 145:833 87:101 143:808 198:311 Reference [10] 66:1	Reference [11] 73.388 132.95 168.89 0·5 1.0 41.233 86.490 92.974 38.462 69.913 96.232 57.277 89.671 129.711 1·0 0·5 43.931 90.950 91.612 39.342 72.274 93.900 55.384 91.015 132.666 0·5 0·5 41.233 86.490 92.974 37.824 72.163 90.455 55.384 91.015 132.666 0·5 0·5 0.5 41.233 86.490 92.974 37.824 72.163 90.455 55.384 91.015 132.666 0·5 0·5 0.5 41.233 86.490 92.974 37.824 72.163 90.455 55.384 91.015 132.666 0·5 0·5 0.5 143.77 29.060 52.530 69.177 42.481 67.298 98.361 Reference [3] 66·18 142.96 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [10] 66·18 142.96 143.775 60.54<	Refe	rence [[5]				73.39	131-7	165.0			
0.5 1.0 41-233 86-490 92-974 38.462 69-913 96-232 57-277 89-671 129-711 1.0 0.5 43-931 90-950 91-612 39-342 72-274 93-900 55-384 91-015 132-666 0.5 0.5 41-233 86-490 92-974 37:824 72-163 90-455 55-195 90-380 125-288 0.5 0.5 31-735 64-892 67-727 29-060 52-530 69-177 42-481 67-298 98:367 1.0 1.0 1.0 66-177 142-740 143-475 60.538 114-565 145-833 87-101 143:808 198:311 Reference [3] 66-18 142-96 143-775 60-54 114-6 145-9 Reference [10] 60-54 114-6 145-9 Reference [5] 60-57 114-7 145-9	0.5 1.0 41.233 86.490 92.974 38.462 69.913 96.232 57.277 89.671 129.711 1.0 0.5 43.931 90.950 91.612 39.342 72.274 93.900 55.384 91.015 132.666 0.5 0.5 41.233 86.490 92.974 37.824 72.163 90.455 55.195 90.280 125.288 0.5 0.5 31.735 64.892 67.727 29.060 52.530 69.177 42.481 67.298 98.367 1.0 1.0 160 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [3] 66.18 142.96 143.775 60.54 114.6 145.9 Reference [5] 66.54 114.7 145.8 87.101 143.808 198.311	Sefe	rence [11]				73·388	132.95	168.89			
1.0 0.5 43.931 90.950 91.612 39.342 72.274 93.900 55.384 91.015 132.666 0.5 0.5 41.233 86.490 92.974 37.824 72.163 90.455 55.195 90.280 125.288 0.5 0.5 31.735 64.892 67.727 29.060 52.530 69.177 42.481 67.298 98.367 1.0 1.0 66.177 142.740 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [3] 66.18 142.96 143.73 60.54 114.565 145.93 87.101 143.808 198.311 Reference [10] 66.18 142.96 143.73 60.54 114.6 145.9 Reference [10] 60.54 114.7 145.9	1.0 0.5 43.931 90.950 91.612 39.342 72.274 93.900 55.384 91.015 132.666 0.5 0.5 41.233 86.490 92.974 37.824 72.163 90.455 55.195 90.280 125.288 0.5 0.5 31.735 64.892 67.727 29.060 52.530 69.177 42.481 67.298 98.367 1.0 1.0 1.0 66.177 142.740 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [3] 66.18 142.96 143.73 60.54 114.6 145.9 Reference [10] 66.18 142.96 143.73 60.54 114.7 145.9 Reference [5] 66.57 114.7 145.9 145.9 145.9		0.5	$1 \cdot 0$	41·233	86.490	92-974	38.462	69-913	96.232	57-277	89.671	129-711
0.5 0.5 41-233 86-490 92·974 37·824 72·163 90·455 55·195 90·280 125·288 0.5 0.5 31·735 64·892 67·727 29·060 52·530 69·177 42·481 67·298 98·367 1.0 1.0 66·177 142·740 143·475 60·538 114·565 145·833 87·101 143·808 198·311 Reference [3] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 66·18 142·96 143·77 60·54 114·6 145·9	0.5 0.5 41-233 86-490 92·974 37.824 72·163 90·455 55·195 90·280 125·288 0.5 0.5 31·735 64-892 67·727 29·060 52·530 69·177 42·481 67·298 98·367 1.0 1.0 1.0 66·177 142·740 143·475 60·538 114·565 145·833 87·101 143·808 198·311 Reference [3] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 60·54 114·6 145·9 60·54 114·7 145·9 Reference [5] 60·57 114·7 145·8 143·9 143·10 143·10		1.0	0.5	43.931	90.950	91.612	39-342	72-274	93-900	55.384	91.015	132.666
0.5 0.5 31.735 64.892 67.727 29.060 52.530 69.177 42.481 67.298 98.367 1.0 1.0 1.0 66.177 142.740 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [3] 66.18 142.96 143.73 60.54 114.6 145.9 Reference [10] 66.18 142.96 143.73 60.54 114.6 145.9 Reference [10] 66.18 142.96 143.73 60.54 114.6 145.9 Reference [10] 66.57 114.7 145.8 145.9 60.57 114.7 145.8	0.5 0.5 0.5 31.735 64.892 67.727 29.060 52.530 69.177 42.481 67.298 98.367 1.0 1.0 1.0 66.177 142.740 143.475 60.538 114.565 145.833 87.101 143.808 198.311 Reference [3] 66.18 143.73 60.54 114.6 145.9 Reference [10] 66.18 143.73 60.54 114.6 145.9 Reference [5] 60.57 114.7 145.8 145.9		0.5	0.5	41.233	86.490	92.974	37-824	72.163	90.455	55.195	90.280	125.288
1.0 1.0 66·177 142·740 143·475 60·538 114·565 145·833 87·101 143·808 198·311 Reference [3] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 66·54 114·7 145·8 145·9	1.0 1.0 1.0 66·177 142·740 143·475 60·538 114·565 145·833 87·101 143·808 198·311 Reference [3] 66·18 142·96 143·73 60·54 114·6 145·9 Reference [10] 60·54 114·6 145·9 60·54 114·7 145·9 Reference [5] 60·57 114·7 145·8 60·54		0.5	0.5	31.735	64.892	67-727	29.060	52.530	69.177	42.481	67-298	98.367
Reference [3] 66·18 142·96 143·73 60·54 Reference [10] 60·54 114·6 145·9 Reference [5] 60·57 114·7 145·8	Reference [3] 66·18 142·96 143·73 60·54 Reference [10] 60·54 114·6 145·9 Reference [5] 60·57 114·7 145·8	_	$1 \cdot 0$	$1 \cdot 0$	66.177	142.740	143-475	60.538	114.565	145.833	87.101	143.808	198.311
Reference [10] 60·54 114·6 145·9 Reference [5] 60·57 114·7 145·8	Reference [10] 60.54 114.6 145.9 Reference [5] 60.57 114.7 145.8	Refe	rence [[3]	66.18	142.96	143.73	60.54					
Reference [5] 60·57 114·7 145·8	Reference [5] 60.57 114.7 145.8	Sefer	rence [10]				60.54	114.6	145-9			
		Refe	rence [[5]				60.57	114-7	145.8			

TRIANGULAR PLATES WITH VARIABLE THICKNESS

$ \begin{bmatrix} 1_3 & \lambda_3 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_2 & \lambda_3 & \lambda_3 & \lambda_4 & \lambda_4 & \lambda_5 & \lambda$	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	λ1
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	16-092
543 57.317 17.766 41.264 60.789 21.774 52.161 7503 758 16.577 42.368 54.560 20.252 50.374 56.115 36 84.432 23.920 62.721 85.744 28.517 71.483 112.432 36 84.432 23.920 62.73 85.744 28.517 71.483 112.432 36 84.432 23.920 62.73 85.77 71.483 112.432 700 45.603 11.139 30.601 47.957 15.386 39.718 66.123 600 45.603 11.025 33.711 45.605 16.045 39.718 62.287 600 45.603 11.025 33.711 45.605 16.045 39.718 62.287 600 45.603 11.025 33.711 45.605 16.045 39.718 62.287 600 45.603 11.025 33.711 45.605 16.045 39.718 62.287 650 68.533 17.316 51.036 72.982 24.327 59.999 98.668 66.330 17.316 51.036 72.982 24.327 59.999 48.668 66.941 17.316 51.04 73.003 12.233 13.538 27.939 48.668 700 68.593 17.316 17.316 51.04 73.003 47.926 47.926 717 11.462 11.715 22.131 36.283 13.538 <td>18-775</td>	18-775
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	18.072
549 $41:503$ $12:945$ $31:651$ $43:325$ $15:386$ $38:404$ $56:115$ $:314$ $84:432$ $23:920$ $62:721$ $85:744$ $28:517$ $71:483$ $112:432$ $:600$ $45:603$ $11:139$ $30:601$ $47:957$ $15:543$ $35:905$ $61:223$ $:600$ $45:603$ $11:139$ $30:601$ $47:957$ $15:543$ $35:905$ $61:223$ $:600$ $45:603$ $11:1025$ $33:711$ $45:605$ $11:025$ $33:711$ $45:603$ $12:033$ $28:065$ $47:666$ $:7:30$ $45:603$ $11:025$ $33:711$ $45:605$ $11:025$ $33:711$ $45:605$ $47:266$ $47:666$ $:7:30$ $45:603$ $11:025$ $33:711$ $45:605$ $12:033$ $28:066$ $47:686$ $:630$ $68:593$ $17:316$ $51:036$ $72:982$ $24:457$ $55:608$ $47:686$ $:7:709$ $68:593$ $17:316$ $51:036$ $72:982$ $24:375$ $59:999$ $95:398$ $:7:709$ $68:593$ $17:316$ $51:036$ $72:983$ $13:558$ $27:939$ $48:668$ $:267$ $41:462$ $11:715$ $22:131$ $36:283$ $13:558$ $27:939$ $48:668$ $:659$ $69:418$ $19:596$ $34:800$ $61:622$ $27:939$ $48:668$ $:669$ $69:418$ $19:596$ $34:800$ $61:622$ $47:752$ $68:597$ $:717$ $11:715$ $22:131$ $36:283$ $13:55202$ $44:752$ $68:597$ <td>17-717</td>	17-717
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	13.707
36 $84\cdot432$ $2.3\cdot93$ 6.77 $85\cdot77$ 600 $45\cdot603$ $11\cdot139$ $30\cdot601$ $47\cdot957$ $15\cdot543$ $35\cdot905$ $61\cdot223$ 720 $43\cdot372$ $12\cdot124$ $33\cdot832$ $50\cdot615$ $18\cdot052$ $40\cdot459$ $64\cdot640$ 600 $45\cdot603$ $11\cdot025$ $33\cdot711$ $45\cdot605$ $16\cdot045$ $39\cdot718$ $62\cdot287$ 600 $45\cdot603$ $11\cdot025$ $33\cdot711$ $45\cdot605$ $16\cdot045$ $39\cdot718$ $62\cdot287$ 700 $45\cdot603$ $11\cdot025$ $33\cdot711$ $45\cdot605$ $16\cdot045$ $39\cdot718$ $62\cdot287$ 700 $45\cdot603$ $17\cdot316$ $51\cdot036$ $72\cdot982$ $24\cdot327$ $59\cdot999$ $95\cdot398$ 700 $68\cdot330$ $17\cdot316$ $51\cdot036$ $72\cdot982$ $24\cdot327$ $59\cdot999$ $95\cdot398$ 700 $68\cdot330$ $17\cdot316$ $51\cdot044$ $73\cdot00$ $72\cdot382$ $27\cdot939$ $48\cdot668$ $77\cdot31$ $77\cdot31$ $51\cdot04$ $73\cdot00$ $16\cdot244$ $27\cdot550$ $47\cdot945$ $73\cdot74$ $14\cdot462$ $11\cdot715$ $22\cdot131$ $36\cdot283$ $13\cdot538$ $27\cdot939$ $48\cdot668$ 980 $41\cdot462$ $11\cdot715$ $22\cdot131$ $36\cdot283$ $13\cdot538$ $27\cdot939$ $48\cdot668$ $73\cdot149$ $96\cdot11$ $32\cdot283$ $13\cdot538$ $27\cdot939$ $48\cdot668$ $73\cdot149$ $93\cdot679$ $37\cdot306$ $17\cdot73$ $49\cdot490$ $16\cdot244$ $27\cdot530$ $47\cdot752$ $669\cdot418$ $19\cdot596$ $34\cdot802$ $61\cdot62$ $27\cdot939$ $48\cdot668$ $77\cdot17$ $19\cdot679$ <td< td=""><td>26.561</td></td<>	26.561
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	C0C-07
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10.518 3
600 $45\cdot603$ $11\cdot025$ $33\cdot711$ $45\cdot605$ $16\cdot045$ $39\cdot718$ $62\cdot287$ $\cdot325$ $32\cdot530$ $7\cdot958$ $24\cdot457$ $35\cdot508$ $12\cdot003$ $28\cdot069$ $47\cdot686$ $\cdot700$ $68\cdot330$ $17\cdot316$ $51\cdot036$ $72\cdot982$ $24\cdot327$ $59\cdot999$ $95\cdot398$ $\cdot700$ $68\cdot593$ $17\cdot316$ $51\cdot036$ $72\cdot982$ $24\cdot327$ $59\cdot999$ $95\cdot398$ $\cdot700$ $68\cdot593$ $17\cdot316$ $51\cdot036$ $72\cdot982$ $24\cdot327$ $59\cdot999$ $95\cdot398$ $\cdot700$ $68\cdot593$ $17\cdot312$ $51\cdot04$ $73\cdot00$ $24\cdot3266$ $47\cdot686$ $\cdot77\cdot52$ $41\cdot462$ $11\cdot715$ $22\cdot131$ $36\cdot283$ $13\cdot538$ $27\cdot939$ $48\cdot668$ $\cdot989$ $41\cdot784$ $14\cdot285$ $18\cdot400$ $40\cdot490$ $16\cdot244$ $27\cdot550$ $47\cdot943$ $\cdot411$ $32\cdot754$ $8\cdot417$ $14\cdot053$ $28\cdot135$ $9\cdot679$ $27\cdot550$ $47\cdot68$ $\cdot559$ $69\cdot418$ $19\cdot566$ $34\cdot802$ $61\cdot107$ $22\cdot646$ $43\cdot965$ $73\cdot514$ $\cdot7117$ $71\cdot033$ $19\cdot61$ $34\cdot802$ $61\cdot107$ $22\cdot646$ $43\cdot965$ $73\cdot514$ $\cdot7117$ $71\cdot033$ $19\cdot61$ $34\cdot802$ $61\cdot62$ $22\cdot302$ $44\cdot752$ $68\cdot597$ $\cdot554$ $67\cdot048$ $21\cdot207$ $42\cdot333$ $60\cdot188$ $25\cdot202$ $44\cdot752$ $68\cdot597$ $\cdot558$ $72\cdot042$ $17\cdot723$ $43\cdot365$ $54\cdot301$ $19\cdot299$ $40\cdot416$ $63\cdot507$ $\cdot528$ $72\cdot042$ $17\cdot723$	10.650 4
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	10.518 34
630 68:330 17:316 51:036 72:982 24:327 59:999 95:338 .709 68:593 17:31 51:04 73:00 95:338 17:53 95:399 95:338 .267 41:462 11:715 22:131 36:283 13:538 27:939 48:668 .267 41:462 11:715 22:131 36:283 13:538 27:939 48:668 .267 41:462 11:715 22:131 36:283 13:538 27:939 48:668 .267 41:784 14:033 18:400 40:490 16:244 27:550 47:943 .411 32:754 8:417 14:053 28:135 9:679 20:228 35:149 .659 69:418 19:596 34:802 61:107 22:646 43:965 73:514 .7117 71:033 19:61 34:8 61:62 27:550 47:752 68:597 .554 67:048 21:207 42:833 60:188 25:202 44:752 68:597 .553 72:042 17:723 43:085 <td>7.283 27</td>	7.283 27
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	16.092 57
	16-092 57-
	13.682 17.
989 41.784 14.285 18.400 40.490 16.244 27.550 47.943 411 32.754 8.417 14.053 28.135 9.679 20.228 35.149 659 69.418 19.596 34.802 61.107 22.646 43.965 73.514 717 71.033 19.61 34.8 61.62 20.228 35.149 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 553 72.042 17.723 43.985 54.301 19.299 40.416 63.506 528 72.042 17.723 43.538 17.808 31.945 51.730 538 101.791 29.093 63.567 89.866 34.641 66.498 99.863 827 101.791 29.09 63.577 89.866 34.641 66.498 99.863 8291 101.85 29.09 63.577 89.866 34.641 66.498	13.682 17.
411 32.754 8.417 14.053 28.135 9.679 20.228 35.149 659 69.418 19.596 34.802 61.107 22.646 43.965 73.514 717 71.033 19.61 34.8 61.62 22.646 43.965 73.514 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 554 67.048 21.207 42.833 60.188 25.202 44.752 68.597 553 72.042 17.723 43.538 17.808 31.945 51.730 827 101.791 29.093 63.567 89.866 34.641 66.498 99.863 891 101.85 29.09 63.57 89.87 89.87 89.87	13.428 16
659 69-418 19-596 34.802 61-107 22-646 43-965 73-514 717 71-033 19-61 34.802 61-107 22-646 43-965 73-514 554 67-048 21-207 34-833 60-188 25-202 44-752 68-597 554 67-048 21-207 42-833 60-188 25-202 44-752 68-597 554 67-048 21-207 42-833 60-188 25-202 44-752 68-597 554 67-048 21-207 42-833 60-188 25-202 44-752 68-597 558 72-042 17-723 43-085 54-301 19-299 40-416 63-206 403 53-809 15-593 32-743 43-538 17-808 31-945 51-730 827 101-791 29-093 63-567 89-866 34-641 66-498 99-863 891 101-85 29-09 63-577 89-87 34-641 66-498 99-863 29-09 63-577 89-87 89-87 89-87	10.220 10.
554 67:048 21:207 42:833 60:188 25:202 44:752 68:597 554 67:048 21:207 42:833 60:188 25:202 44:752 68:597 554 67:048 21:207 42:833 60:188 25:202 44:752 68:597 528 72:042 17:723 43:085 54:301 19:299 40:416 63:206 403 53:809 15:593 32:743 43:538 17:808 31:945 51:730 827 101:791 29:093 63:567 89:866 34:641 66:498 99:863 891 101:85 29:09 63:57 89:87 34:641 66:498 99:863	22.646 26. 22.666 26.
554 67.048 21-207 42.833 60-188 25-202 44.752 68:597 554 67.048 21-207 42.833 60-188 25-202 44.752 68:597 528 72.042 17.723 43.085 54.301 19.299 40.416 63.206 403 53.809 15.593 32.743 43.538 17.808 31.945 51.730 827 101.791 29.093 63.567 89.866 34.641 66.498 99.863 891 101.85 29.09 63.57 89.806 34.641 66.498 99.863	
-554 67.048 21.207 42.833 60.188 25.202 44.752 68:597 -528 72.042 17.723 43.085 54.301 19.299 40.416 63.206 -403 53.809 15:593 32.743 43.538 17.808 31.945 51.730 -827 101.791 29.093 63.567 89.866 34.641 66.498 99.863 -891 101.85 29.09 63.57 89.816 34.641 66.498 99.863	29.928 65
.528 72.042 17.723 43.085 54.301 19.299 40.416 63.206 .403 53.809 15.593 32.743 43.538 17.808 31.945 51.730 .827 101.791 29.093 63.567 89.866 34.641 66.498 99.863 .891 101.85 29.09 63.57 89.87	29-928 65
-403 53.809 15.593 32.743 43.538 17.808 31.945 51.730 -827 101.791 29.093 63.567 89.866 34.641 66.498 99.863 -891 101.85 29.09 63.57 89.87 29.87	27-576 60
-827 101·791 29·093 63·567 89·866 34·641 66·498 99·863 -891 101·85 29·09 53·57 89·87 29·09 63·57 89·87	23.610 48
-891 101-85 29-09 53-57 89-87 29-09	40.016 9
29-09 63-57 89-87	40.022 9

B. SINGH AND S. M. HASSAN

c			۳ مرب د	$= 0.5, \ \eta = \sqrt{3}$	1 ABLE 3/2		$= 0.0, \eta = 1$		اا مربد ر	$-0.5, \eta = $	3/2
$\beta \gamma \lambda_1$	$\gamma = \lambda_1$	λ_1		λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_{3}
Reference Ours Reference [3] Reference [7]	urs 3]					28-872 28-87 28-87	62.969 62.97 62.97	89-671 89-66 80-65			
5 0.5 1.0 17.717 48 ⁻	1.0 17.717 48.	17.717 48.	48.	758	55.183	12.157	30.471	49.452	13.806	30.193	49.310
5 1.0 0.5 18.775 49.7	0.5 18.775 49.7	18.775 49.7	49.7	125	56.531	12.740	31.722	49.321	13.398	31.431	52.439
0 0.5 0.5 18.072 49.5	0.5 18.072 49.5	18.072 49.5	49.5	43	57.317	11.059	32.739	44.954	11.226	29.555	48.652
5 0.5 0.5 13.707 38.54	0.5 13.707 38.54	13.707 38.54	38.54	6†	$41 \cdot 503$	8.846	23.525	35-903	9.591	22.241	37-986
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	1.0 26.561 75.31	26.561 75.31	75.31		84.350	17.967	47-949	73-629	18-951	46.791	75-941
Reference [10]	00.07 [0	06.01 000.07	06.67		764.40	17.97	47.95	73.63			
5 0.5 1.0 5.247 23.171	1.0 5.247 23.171	5.247 23.171	23.171		25.440	3.963	15.194	24.638	3.968	15-355	26.680
5 1.0 0.5 6.032 24.429	0.5 6.032 24.429	6.032 24.429	24.429		24·470	4.115	15.673	21.668	3.617	14.430	24.645
0 0.5 0.5 5.247 23.171	0.5 5.247 23.171	5.247 23.171	23.171		25-440	3.670	14.737	21.358	3.579	13.363	23.239
5 0.5 0.5 3.872 17.776	0.5 3.872 17.776	3·872 17·776	17.776		17-943	2.796	11.069	16.890	2.681	10.854	18.907
0 1.0 1.0 8.921 35.095	1.0 8.921 35.095	8-921 35-095	35.095		38-484	$6 \cdot 167$	23.458	32.682	5.701	21.501	36.269
Reference [3] 8.9219 35.155	3] 8-9219 35-155	8-9219 35-155	35.155		38.503	6.173					
Reference [9] 8.9221 35.132	9] 8-9221 35-132	8-9221 35-132	35.132		38.505	6.1732	23-477	32.716			
Reference [10]	0]					6.168	23-46	32.69			
Reference [14]	4]					5.93	23-4	32.7			
Reference [13]	3]					6.16	23.7	32.54			
Reference [12]	2]					6.1215	23.02	31.853			
Reference [11]	1]					6.1575	23.436	32.735			
Reference [16]	[9]					6.157	23.415	32.621			
5 0.5 1.0 18.775 49.725	1.0 18.775 49.725	18.775 49.725	49.725		56.531	12.740	31.722	49.321	13.398	31-431	52.439
5 1.0 0.5 17.717 48.758	0.5 17.717 48.758	17.717 48.758	48·758		55.183	12.157	30.471	49.452	13.806	30.193	49.310
0 0.5 0.5 18.072 49.543	0.5 18.072 49.543	18.072 49.543	49.543		57.317	11.059	32.739	44.954	11.226	29.555	48.652
5 0.5 0.5 13.707 38.549	0.5 13.707 38.549	13.707 38.549	38.549		41.503	8.846	23.525	35-903	9.591	22.241	37-986
0 1.0 1.0 26.561 75.314	1.0 26.561 75.314	26.561 75.314	75.314		84.350	17.967	47-949	73.629	18.951	46.791	75-941
Reference [3] 26·565 75·36	3] 26.565 75.36	26.565 75.36	75.36		84.432	17.96					
Reference [10]	0]					17.97	47.95	73.63			
										contin	ned overled

TABLE 2—continued

TRIANGULAR PLATES WITH VARIABLE THICKNESS

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$							I ABLE	2-continu	per				
\vec{x} \vec{p} $\vec{\lambda}_1$ $\vec{\lambda}_2$ $\vec{\lambda}_1$ $\vec{\lambda}_2$ $\vec{\lambda}_1$ $\vec{\lambda}_2$ $\vec{\lambda}_3$ $\vec{\lambda}_4$ <t< th=""><th>- 6</th><th></th><th></th><th></th><th>π 1</th><th>= 0.5, $\eta =$</th><th>3/2</th><th>ž</th><th>$= 0.0, \eta = 1$</th><th>0.</th><th>ζ = .</th><th>$-0.5, \eta =$</th><th>/3/2</th></t<>	- 6				π 1	= 0.5 , $\eta = $	3/2	ž	$= 0.0, \eta = 1$	0.	ζ = .	$-0.5, \eta = $	/3/2
$ \begin{array}{l c c c c c c c c c c c c c c c c c c c$	1 ლ	8	β	γ	λ1	λ_2	λ3	λ1	λ_2	λ_3	21	λ_2	λ_3
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ĹĿ	0.5	0.5	$1 \cdot 0$	10.518	34.600	45.603	7.136	20.750	38-573	7.778	18.579	35.476
5 1:0 0:5 0:5 1:0 0:5 0:5 1:0 0:5 0:5 1:0 0:5	7	0.5	$1 \cdot 0$	0.5	10.518	34.600	45.603	7.136	20.750	38-573	7.778	18.579	35.476
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		$1 \cdot 0$	0.5	0.5	10.650	40.720	43.372	6.051	24.248	35.750	5.912	20.161	36.578
		0.5	0.5	0.5	7.283	27.325	32.530	5.023	15.383	28-076	6.150	13.168	26.593
$ \begin{array}{llllllllllllllllllllllllllllllllllll$		1.0	$1 \cdot 0$	$1 \cdot 0$	16.092	57-630	68.330	9.798	34.633	58.574	8-974	30.652	55.214
Reference [10] 9.798 34.63 58.56 5 0.5 1.0 13.682 17.567 41.462 9.366 17.641 25.060 8.101 18.021 28.633 5 0.5 1.9 13.482 17.567 41.462 9.366 17.641 25.060 8.101 18.021 28.633 6.5 0.5 0.5 13.682 17.567 41.462 9.366 17.641 25.766 19.383 25.863 29.938 19.933 27.881 27.881 27.881 27.881 27.881 27.733 27.735 41.905 7.881 27.775 41.905 7.861 17.843 27.175 41.905 7.861 17.863 29.63 19.963 19.955 19.955 27.775 41.905 7.861 17.863 29.63 27.175 41.905 7.745 7.745 7.745 7.745 7.745 7.745 7.745 7.463 2.7617 21.609 37.455 7.668 27.617 21.993 27.455		Re	ference	[3]	16.092	57.709	68.593	9.798					
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Ref	erence [[<u>1</u> 0]				9.798	34.63	58-56			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ſт	0.5	0.5	$1 \cdot 0$	13.682	17-267	41.462	9.366	17-641	25.060	8.101	18.021	28.630
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	~	0.5	$1 \cdot 0$	0.5	13.428	16.989	41.784	8.266	16.735	24.869	6.499	17.853	25.874
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ſŦ.	1.0	0.5	0.5	13.682	17.267	41.462	9.139	14-491	29.753	7.076	17.329	27.581
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.5	0.5	0.5	10.220	10.411	32.754	6.191	11.212	18.738	5.130	12.956	19.388
$ \begin{array}{llllllllllllllllllllllllllllllllllll$		1.0	$1 \cdot 0$	$1 \cdot 0$	22.646	26.659	69.418	14.561	24.738	42.039	11.380	27.175	41.995
$ \begin{array}{l c c c c c c c c c c c c c c c c c c c$		Re	ference	[3]	22.666	26.717	71.033	14.56					
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Ref	erence [[10]				14.56	24·74	42.07			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ſŦ.	0.5	0.5	$1 \cdot 0$	6.032	24-429	24.470	4.115	15.673	21.668	3.617	14.430	24.645
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ſт.	0.5	1.0	0.5	5.247	23.171	25.440	3.963	15.194	24.638	3.968	15.355	26.680
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	٢)	$1 \cdot 0$	0.5	0.5	5.247	23.171	25.440	3.670	14.737	21.358	3.579	13.363	23·239
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.5	0.5	0.5	3.872	17.776	17-943	2.796	11.069	16.890	2.681	10.854	18.907
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$1 \cdot 0$	$1 \cdot 0$	$1 \cdot 0$	8.921	35.095	38-484	6.167	23-458	32.682	5.701	21.501	36.269
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Re	ference	[1]				6.1575	23.059	33.288	5.7617	21.099	35.952
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Re	ference	[3]	8-9219	35.155	38.503	$6 \cdot 173$			5.7167	21.524	37-456
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Re	ference	[6]	8.9221	35.132	38.505				5.717	21.525	37-455
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		Rei	erence	[10]				$6 \cdot 168$	23-46	32.69			
$ \begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$	ĹL	0.5	0.5	$1 \cdot 0$	13.428	16.989	41.784	8.266	16.735	24.869	6.499	17.853	25.874
S 1:0 0:5 0:5 13:682 17:267 41:462 9:139 14:491 29:753 7:076 17:329 27:581 0:5 0:5 0:5 10:220 10:411 32:754 6:191 11:212 18:738 5:130 12:956 19:388 1:0 1:0 1:0 22:646 26:659 69:418 14:561 24:738 42:039 11:380 27:175 41:995 Reference [3] 22:666 26:717 71:033 14:56 24:74 42:07 Reference [10] 12:22:666 26:717 71:033 14:56 24:74 42:07	ĹĿ	0.5	$1 \cdot 0$	0.5	13.682	17.267	41.462	9.366	17.641	25.060	8.101	18.021	28.630
0.5 0.5 0.5 0.5 10.220 10.411 32.754 6.191 11.212 18.738 5.130 12.956 19.388 1.0 1.0 1.0 22.646 26.659 69.418 14.561 24.738 42.039 11.380 27.175 41.995 Reference [3] 22.666 26.717 71.033 14.56 24.74 42.07 Reference [10] 14.56 24.74 42.07	S	$1 \cdot 0$	0.5	0.5	13.682	17.267	41.462	9.139	14-491	29.753	7-076	17.329	27.581
1.0 1.0 1.0 22.646 26.659 69.418 14.561 24.738 42.039 11.380 27.175 41.995 Reference [3] 22.666 26.717 71.033 14.56 24.74 42.039 11.380 27.175 41.995 Reference [10] 1 14.56 24.74 42.07		0.5	0.5	0.5	10.220	10.411	32.754	$6 \cdot 191$	11.212	18.738	5.130	12.956	19.388
Reference [3] 22.666 26.717 71.033 14.56 Reference [10] 14.56 24.74 42.07		$1 \cdot 0$	$1 \cdot 0$	$1 \cdot 0$	22.646	26.659	69.418	14.561	24.738	42.039	11.380	27.175	41.995
Reference [10] 14·56 24·74 42·07		Re	ference	[3]	22.666	26.717	$71 \cdot 033$	14.56					
		Rei	erence	[10]				14.56	24·74	42.07			

B. SINGH AND S. M. HASSAN

	3/2	λ_3	27.588	27.588	31-437	19.761	46.639			
	$-0.5, \eta = $	λ_2	16.331	16.331	16.643	11.018	25.220			
	ς ε	λ1	7.592	7.592	8-496	5.522	13-434			
	0	λ_3	28.727	28.727	$23 \cdot 101$	18.615	45.450		46.03	
ed	$= 0.0, \eta = 1.$	λ_2	18.258	18-258	18-753	11.933	29.129		29-25	
2-continu	ນັກ	λ,	10.211	10.211	12.340	7.523	19.072	19.17	19-08	
TABLE	3/2	λ_3	23-926	23.926	23.926	14.611	36.072	36-337		
	= $0.5, \eta = $	λ_2	$23 \cdot 100$	$23 \cdot 100$	$23 \cdot 100$	14.611	36.072	36.331		
	اا س	, 71	16.842	16.842	16.842	12.881	34·283	34-962		
		λ	$1 \cdot 0$	0.5	0.5	0.5	$1 \cdot 0$	[3]	[10]	
		β	0.5	$1 \cdot 0$	0.5	0.5	$1 \cdot 0$	ference	erence	
		ø	0.5	0.5	$1 \cdot 0$	0.5	$1 \cdot 0$	Re	Ref	
		1 ന	Ц	Ľ	ц					

TABLE 2—continued

123 x_1 λ_3 λ_4 λ_4 λ_5 λ_4 λ_5 λ_4 λ_5 λ_4 λ_5 λ_4 λ_5 λ_5 λ_4 λ_5			ж,	$= 0.5, \ \eta = \sqrt{3}$	3/2	чл	$= 0.0, \eta = 1.$	0	ا س	$-0.5, \eta = $	/3/2
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	123	8	אן	λ_2	λ_3	у,	λ_2	λ_3	۲ ^ا	h2	λ_3
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CCC	$\begin{array}{c} 0.5\\ 1.5\end{array}$	82·271 114·742	146·797 226·112	146·797 226·112	78-082 108-565	124·327 188·103	152·642 232·175	117.376 161.874	169-781 245-717	226-417 319-652
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CCS	0.5 1.5	65·572 97·241	124·219 199·186	128.907 201.379	63·757 93·404	107.620 167.854	136·221 208·470	95-414 140-163	146-486 219-253	200-893 287-931
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CCF	0.5 1.5	34·642 45·362	78·607 111·354	81·352 125·315	35·588 46·622	70·614 101·935	90.054 130.335	49·254 67·215	92·280 129·280	128·273 183·897
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CSC	0.5 1.5	65·572 97·241	124·219 199·186	128.907 201.379	63·757 93·404	107·620 167·854	136·221 208·470	95·414 140·163	146·485 219·253	200-892 287-932
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CSS	0.5 1.5	51·259 81·538	104.807 175.312	110-298 177-462	51·309 79·861	91·584 149·175	120.933 186.607	77-635 121-564	123·667 194·652	177·544 262·148
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CSF	0.5 1.5	21·551 31·754	62·068 89·739	63.324 106.409	26·041 37·208	57·267 87·953	77-028 113-931	41-956 59-569	74·759 114·117	111-256 163-445
$ \begin{array}{rcccccccccccccccccccccccccccccccccccc$	CFC	0.5 1.5	34·642 45·362	78·607 111·354	81·352 125·315	35·588 46·622	70·614 101·935	90.054 130.335	49·254 67·215	92·279 129·280	128·271 183·896
CFF 0.5 6:562 26:995 30:469 9:037 27:960 41:818 15:650 38:528 649 1:5 10:946 44:024 47:238 15:669 44:531 64:577 29:131 62:123 95:6 SCC 0:5 65:572 124:219 128:907 57:667 101:790 123:247 81:469 129:404 179:7 SCC 0:5 51:259 109:186 201:379 88:823 160:244 201:883 129:404 179:7 SCS 0:5 51:259 104:807 110:298 46:050 86:875 108:176 64:366 109:868 157:2 SCF 0:5 51:259 104:807 110:298 46:050 86:875 108:176 64:366 109:868 157:2 SCF 0:5 51:551 62:068 63:324 141:876 180:122 110:046 179:306 241:6 SCF 0:5 51:551 62:068 63:324 19:386 57:33	CFS	0.5 1.5	21·551 31·754	62·068 89·739	63.324 106.409	26·041 37·208	57·267 87·953	77-028 113-931	41-956 59-569	74·759 114·117	111.256 163.445
SCC 0.5 65:572 124:219 128:907 57:667 101:790 123:247 81:469 129:404 179:7 1:5 97:241 199-186 201:379 88:823 160:244 201:883 128:880 202:986 269:1 SCS 0.5 51:259 104-807 110:298 46:050 86:875 108:176 64:366 109:868 157:2 SCS 0.5 51:259 104-807 110:298 46:050 86:875 108:176 64:366 109:868 157:2 SCF 0.5 21:551 62:068 63:324 19:386 50:251 65:960 22:735 57:872 87:5 SCF 0.5 51:754 89:739 106:409 29:024 75:438 107:498 35:293 85:599 138:3 SSC 0.5 51:259 104:402 29:024 75:438 107:498 35:293 85:599 138:3 SSC 0.5 51:253 108:176 64:366 <td< td=""><td>CFF</td><td>0.5 1.5</td><td>6.562 10.946</td><td>26·995 44·024</td><td>30-469 47-238</td><td>9-037 15-669</td><td>27-960 44-531</td><td>41·818 64·577</td><td>15·650 29·131</td><td>38·528 62·123</td><td>64-948 95-652</td></td<>	CFF	0.5 1.5	6.562 10.946	26·995 44·024	30-469 47-238	9-037 15-669	27-960 44-531	41·818 64·577	15·650 29·131	38·528 62·123	64-948 95-652
SCS 0.5 51-259 104-807 110-298 46·050 86·875 108·176 64·366 109·868 157·2 1-5 81·538 175·312 177·462 75·473 141·876 180·122 110·046 179·306 241·0 SCF 0·5 21·551 62·068 63·324 19·386 50·251 65·960 22·735 57·872 87·5 SCF 0·5 21·551 62·068 63·324 19·386 50·251 65·960 22·735 57·872 87·5 SCF 0·5 31·754 89·739 106·409 29·024 75·438 107·498 35·293 85·599 138·3 SSC 0·5 51·259 104·807 110·298 46·050 86·875 108·176 64·366 109·868 157·2 SSC 0·5 81·538 177·462 75·473 141·876 180·122 110·046 179·306 241·6	SCC	0.5 1.5	65·572 97·241	124·219 199·186	128-907 201-379	57·667 88·823	101-790 160-244	123·247 201·883	81.469 128.880	129-404 202-986	179-783 269-128
SCF 0.5 21:551 62:068 63:324 19:386 50:251 65:960 22:735 57:872 87:5 1:5 31:754 89:739 106:409 29:024 75:438 107:498 35:293 85:599 138:3 SSC 0.5 51:259 104:807 110:298 46:050 86:875 108:176 64:366 109:868 157:2 I:5 81:538 177:462 75:473 141:876 180:122 110:046 179:306 241:6	SCS	0.5 1.5	51·259 81·538	104-807 175-312	110-298 177-462	46·050 75·473	86-875 141-876	108·176 180·122	64.366 110.046	109-868 179-306	157-291 241-056
SSC 0·5 51·259 104·807 110·298 46·050 86·875 108·176 64·366 109·868 157·2 1·5 81·538 175·312 177·462 75·473 141·876 180·122 110·046 179·306 241·6	SCF	0.5 1.5	21·551 31·754	62·068 89·739	63.324 106.409	19-386 29-024	50·251 75·438	65.960 107.498	22·735 35·293	57·872 85·599	87-556 138-330
	SSC	0.5 1.5	51·259 81·538	104.807 175.312	110-298 177-462	46-050 75-473	86-875 141-876	108·176 180·122	64.366 110.046	109-868 179-306	157-291 241-056

TABLE 3

B. SINGH AND S. M. HASSAN

		اا س	$= 0.5, \ \eta = \sqrt{3}$	3/2	w	$= 0.0, \eta = 1.$	0	یں ا	$-0.5, \eta = $	3/2
123	8	л. Л	N2	λ3	ץ'	Å2	λ_3	۲'	λ_2	λ_3
SSS	$0.5 \\ 1.5$	39-064 67-620	89-929 154-373	89-929 154-373	35-929 63-717	72-736 125-028	94-237 160-161	50·244 94·128	90-372 157-623	137-980 216-816
SSF	0.5 1.5	12·071 20·737	44·745 71·665	49-927 88-555	13-020 22-076	38·794 63·866	55-076 93-015	18.665 30.612	44·641 74·741	73·169 120·230
SFC	$\begin{array}{c} 0.5\\ 1.5\end{array}$	21.551 31.754	62·068 89·739	63·324 106·409	19·386 29·024	50·251 75·438	65.960 107.498	22·735 35·293	57·872 85·599	87·557 138·330
SFS	$\begin{array}{c} 0.5\\ 1.5\end{array}$	12·071 20·737	44·745 71·665	49-927 88-555	13-020 22-076	38·794 63·866	55-076 93-015	18.665 30.612	44·641 74·741	73·169 120·230
SFF	0.5 1.5	16·750 29·417	18.627 35.900	53·160 86·044	14.131 26.090	24·650 46·007	44·388 79·198	16.077 30.615	33·371 56·387	57·219 90·744
FCC	$\begin{array}{c} 0.5\\ 1.5\end{array}$	34·642 45·362	78·607 111·354	81·352 125·315	24·472 33·020	52·482 75·322	68-430 109-881	28·479 38·991	53·788 79·207	81.966 118.696
FCS	$\begin{array}{c} 0.5\\ 1.5\end{array}$	21.551 31.754	62·068 89·739	63·324 106·409	14·327 21·528	38·357 58·430	55·463 91·124	15·070 22·601	36·929 57·253	61·151 92·027
FCF	$\begin{array}{c} 0.5\\ 1.5\end{array}$	6.562 10.946	26·995 44·024	30-469 47-238	4·579 7·607	18·372 28·811	25·632 40·417	4·227 7·230	17·310 25·727	28·882 43·832
FSC	0.5 1.5	21.551 31.754	62·068 89·739	63·324 106·409	14·327 21·528	38·357 58·430	55·463 91·124	15·070 22·601	36-929 57-253	61·151 92·027
FSS	$\begin{array}{c} 0.5\\ 1.5\end{array}$	12·071 20·737	44·745 71·665	49-927 88-555	7·721 12·429	25·866 44·619	43·833 73·605	7-920 10-661	22·343 40·397	43·314 69·124
FSF	0.5 1.5	16·750 29·417	18.627 35.900	53·160 86·044	10.562 19.332	18.448 31.487	31·173 54·572	8-426 15-035	21·369 33·445	30·365 55·805
FFC	0.5 1.5	6.562 10.946	26·995 44·024	30-469 47-238	4·579 7·607	18·372 28·811	25·632 40·417	4·227 7·230	17·310 25·727	28·882 43·832
FFS	0.5 1.5	16·750 29·417	18.627 35.900	53·160 86·044	10.562 19.332	18-448 31-487	31·173 54·572	8-426 15-035	21·369 33·445	30·365 55·805
FFF	0.5 1.5	23·804 45·646	25-050 49-317	25-050 49-317	13·384 25·850	20-718 38-601	32-038 60-052	$9.561 \\ 18.169$	19-234 31-916	32·240 56·700

TABLE 3—continued

TRIANGULAR PLATES WITH VARIABLE THICKNESS

TABLE	4
TIDDD	

	genee of	resuits	joi quaa	ane mich	ness curi	(115	<i>ure</i> 5(e),	$\alpha = 0.5,$	p = 0.5
123	ξ	η	N = 10	15	21	28	34	35	36
CCC	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\sqrt{3}/2$	50·860 50·102 80·088	50·354 50·669 79·449	50·152 50·529 79·002	50·068 50·486 78·827	50·038 50·472 78·804	50·032 50·472 78·794	50·032 50·470 78·793
CCS	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\frac{\sqrt{3}/2}{\sqrt{3}/2}$	44·273 45·492 72·189	43·790 45·027 70·946	43·619 44·886 70·526	43·554 44·846 70·276	43·534 44·837 70·207	43·529 44·835 70·203	43·528 44·834 70·201
CCF	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\\sqrt{3}/2$	28·228 29·847 42·389	28·176 29·788 41·773	28·147 29·769 41·432	28·140 29·757 41·334	28·137 29·750 41·267	28·137 29·750 41·260	28·137 29·750 41·252
CSC	$0.5 \\ 0.0 \\ -0.5$	$\sqrt{3/2} \ 1.0 \ \sqrt{3/2} \ \sqrt{3/2}$	41·478 39·787 59·468	40·486 38·994 58·391	40·041 38·685 57·556	39·836 38·547 57·349	39·738 38·489 57·264	39·738 38·488 57·235	39·737 38·486 57·234
CSS	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\\sqrt{3}/2$	35·666 35·318 54·090	34·699 34·476 52·886	34·310 34·169 52·121	34·147 34·042 51·849	34·077 33·993 51·740	34·076 33·993 51·731	34·075 33·990 51·730
CSF	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\\sqrt{3}/2$	17·900 21·112 34·955	17·819 21·004 33·723	17·794 20·974 33·717	17·785 20·965 33·622	17·781 20·962 33·606	17·781 20·962 33·606	17·781 20·961 33·606
CFC	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\\sqrt{3}/2$	19·578 17·402 21·427	18·848 16·741 20·526	18·403 16·341 19·930	18·135 16·100 19·618	17·963 15·942 19·425	17·960 15·940 19·423	17·959 15·939 19·423
CFS	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\\sqrt{3}/2$	11.856 13.392 20.019	11·418 12·806 18·970	11·199 12·505 18·359	11.079 12.328 18.003	11·011 12·227 17·802	11·010 12·227 17·800	11·010 12·225 17·797
CFF	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}\\ \sqrt{3}/2$	3·930 5·029 8·461	3·860 4·917 8·126	3·838 4·868 8·017	3·830 4·846 7·964	3·826 4·834 7·941	3·826 4·834 7·941	3·826 4·834 7·941
SCC	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\frac{\sqrt{3}/2}{\sqrt{3}/2}$	43·047 42·685 67·399	42·752 42·367 65·497	42·644 42·274 65·396	42·596 42·250 65·145	42·581 42·243 65·128	42·579 42·242 65·121	42·577 42·241 65·118
SCS	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\frac{\sqrt{3}/2}{\sqrt{3}/2}$	36·818 37·194 59·434	36·509 36·921 57·353	36·413 36·820 57·130	36·374 36·799 56·851	36·363 36·792 56·774	36·362 36·792 56·769	36·360 36·791 56·766
SCF	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\frac{\sqrt{3}/2}{\sqrt{3}/2}$	18·873 18·619 23·706	18·819 18·578 23·108	18·801 18·568 22·821	18·797 18·557 22·693	18·795 18·551 22·624	18·795 18·551 22·621	18·795 18·551 22·618
SSC	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\frac{\sqrt{3}/2}{\sqrt{3}/2}$	34·752 32·450 47·779	34·115 32·213 47·256	33·914 32·064 46·837	33·830 32·015 46·623	33·799 31·997 46·578	33·793 31·995 46·578	33·793 31·994 46·578
SSS	$0.5 \\ 0.0 \\ -0.5$	$ \sqrt{3/2} \\ 1 \cdot 0 \\ \sqrt{3/2} $	29·310 28·163 42·632	28.656 27.826 41.845	28·464 27·700 41·510	28·393 27·652 41·310	28·370 27·637 41·229	28·365 27·634 41·227	28·365 27·634 41·226
SSF	$0.5 \\ 0.0 \\ -0.5$	$\frac{\sqrt{3}/2}{1\cdot 0}$ $\sqrt{3}/2$	10·615 12·449 19·239	10·572 12·410 18·974	10·569 12·406 18·840	10·568 12·405 18·801	10·568 12·405 18·766	10·568 12·405 18·766	10·568 12·405 18·766

Convergence of results for quadratic thickness variation (Figure 3(c), $\alpha = 0.3$, $\beta = 0.5$)

continued overleaf

_					THELL	continu				
_	123	ξ	η	N = 10	15	21	28	34	35	36
-	SFC	0.5	$\sqrt{3}/2$	15.961	15.752	15.663	15.613	15.588	15.588	15.586
		0.0	1.0	13.715	13.610	13.545	13.507	13.490	13.490	13.486
		-0.5	$\sqrt{3/2}$	16.262	16.185	16.038	15.963	15.943	15.940	15.939
	SFS	0.5	$\sqrt{3}/2$	9.151	9.077	9.037	9.019	9.010	9.010	9.010
		0.0	·1 <u>·</u> 0	10.000	9.908	9.870	9.848	9.837	9.837	9.835
		-0.5	$\sqrt{3/2}$	14.480	14.348	14.292	14.229	14.212	14.199	14.197
	SFF	0.5	$\sqrt{3}/2$	11.620	11.136	11.016	10.947	10.918	10.916	10.914
		0.0	1 <u>-</u> 0	10.144	9.990	9.921	9.882	9.867	9.862	9.860
		-0.5	$\sqrt{3/2}$	12.580	12.526	12.391	12.341	12.330	12.323	12.323
	FCC	0.5	$\sqrt{3/2}$	22.509	22.395	22.372	22.361	22.360	22.359	22.356
		0.0	1.0	20.207	20.099	20.055	20.047	20.047	20.047	20.047
		-0.5	$\sqrt{3}/2$	29.321	28.630	28.566	28.500	28.495	28.494	28.489
	FCS	0.5	$\sqrt{3/2}$	14.102	14.077	14.071	14.067	14.067	14.067	14.066
		0.0	1 <u>·</u> 0	12.106	12.056	12.044	12.044	12.044	12.044	12.044
		-0.5	$\sqrt{3/2}$	15.844	15.432	15.234	15.141	15.092	15.092	15.092
	FCF	0.5	$\sqrt{3}/2$	4.852	4.843	4.840	4.839	4.839	4.839	4.838
		0.0	1.0	4.189	4.165	4.155	4.149	4.146	4.146	4.146
		-0.5	$\sqrt{3/2}$	4.302	4.249	4.226	4.217	4.212	4·212	4.211
	FSC	0.5	$\sqrt{3}/2$	18.211	18.006	17.961	17.949	17.946	17.945	17.945
		0.0	1.0	14.633	14.606	14.595	14.590	14.590	14.589	14.589
		-0.5	$\sqrt{3/2}$	17.813	17.513	17.455	17.403	17.385	17.385	17.385
	FSS	0.5	$\sqrt{3}/2$	10.766	10.694	10.684	10.681	10.681	10.680	10.680
		0.0	1.0	8.319	8.315	8.314	8.313	8.313	8.313	8.313
		-0.5	$\sqrt{3}/2$	10.038	9.761	9.597	9.484	9.407	9.407	9.407
	FSF	0.5	$\sqrt{3}/2$	14.031	13.730	13.581	13.546	13.540	13.540	13.538
		0.0	1_0	10.585	10.521	10.498	10.495	10.494	10.493	10.493
		-0.5	$\sqrt{3/2}$	9.483	9.440	9.404	9.394	9.386	9.386	9.385
	FFC	0.5	$\sqrt{3}/2$	10.197	10.143	10.132	10.127	10.126	10.126	10.126
		0.0	1_0	7.235	7.228	7.218	7.216	7.215	7.215	7.215
		-0.5	$\sqrt{3/2}$	6.538	6.493	6.485	6.478	6.475	6.475	6.475
	FFS	0.5	$\sqrt{3}/2$	16.467	15.535	15.298	15.217	15.198	15.188	15.188
		0.0	1 <u>-0</u>	10.572	10.401	10.342	10.331	10.329	10.328	10.328
		-0.5	$\sqrt{3/2}$	8.873	8.726	8.713	8.710	8.709	8.708	8.708
	FFF	0.5	$\sqrt{3}/2$	22.228	18.432	17.276	16.886	16.784	16.780	16.765
		0.0	1 <u>·0</u>	14.292	11.988	11.674	11.572	11.556	11.556	11.554
		-0.5	$\sqrt{3/2}$	10.852	9.536	9.362	9.351	9.345	9.345	9.343

TABLE 4—continued

B. SINGH AND S. M. HASSAN

said that four digits are significant and in the last we have three digits, namely, 1, 2 and 8, agreeing with the exact result. This particular case serves as a test of the accuracy of our results.

The effects of the various parameters, α , β , γ , α' , β' , and the boundary conditions are self-explanatory from Tables 1 through 4. A general observation can be made that as the overall thickness increases, the frequency also increases. Further, as one goes from all free edges to all simply-supported edges and then to all clamped edges, the frequencies increase. It is difficult to comment when the boundary conditions are mixed since some have increasing, while others decreasing, effects on the frequency.

4. MODE SHAPES

Figure 4 gives the first three mode shapes of an equilateral triangle with all sides clamped and uniform thickness (Table 2, CCC, $\alpha = \beta = \gamma = 1$, $\xi = 0.5$, $\eta = \sqrt{3}/2$). Figure 5 gives the first three mode shapes of an isosceles triangle with angles of 30, 30 and 120°, again with all sides clamped but thickness varying quadratically (Table 1, CCC, $\alpha = 0.3$, $\beta = 0.5$, $\xi = -0.5$, $\eta = \sqrt{3}/2$). Figure 6 gives mode shapes for the same triangle as in Figure 5 but sides facing vertices 1, 2 and 3 are simply-supported, clamped and free, respectively. The thickness varies quadratically (Table 2, SCF, $\alpha = 0.5$, $\beta = 1.0$, $\gamma = 0.5$, $\xi = -0.5$, $\eta = \sqrt{3}/2$). For want of space, only a few selected cases have been given here but the program can generate mode shapes for a triangle of any shape and arbitrary thickness variation which can be approximated up to a cubic variation and for any combination of boundary conditions.



Figure 4. First three mode shapes of an equilateral triangle with uniform thickness, CCC.



Figure 5. First three mode shapes of an isosceles triangle with angles 30, 30 and 120° and quadratic thickness variation, CCC.



Figure 6. First three mode shapes of isosceles triangle with angles 30, 30 and 120° and quadratic thickness variation, SCF.

B. SINGH AND S. M. HASSAN

5. CONCLUSION

The present method gives numerical results for frequency and mode shapes of practically any triangular plate with arbitrary thickness variation and any combination of boundary conditions. The crux of the matter lies in approximating the thickness variation by a polynomial of suitable degree. This can be done by choosing a set of sample points and carrying out measurements for thickness at these points and fitting a polynomial by interpolation. The convergence is ensured by working out a large number of approximations using the Rayleigh–Ritz method and suitable basis functions satifying the essential boundary conditions. The use of natural co-ordinates has simplified the computations to a great extent.

ACKNOWLEDGMENTS

We wish to thank the Faculty of Science, Ain Shams University, Cairo, Egypt for providing financial support to S.M.H. Thanks are also due to All India Council of Technical Education, New Delhi for providing financial help in a project to B.S. We also wish to thank Central Computational Facility and the Computational Laboratory staff where part of the work was carried out.

REFERENCES

- 1. S. MIRZA and M. BIJLANI 1985 *International Journal of Computers and Structures* 21, 1129–1135. Vibration of triangular plates of variable thickness.
- 2. B. SINGH and V. SAXENA 1996 *Journal of Sound and Vibration* **194**, 471–496. Transverse vibration of triangular plates with variable thickness.
- 3. B. SINGH and S. CHAKRAVERTY 1992 *International Journal of Mechanical Sciences* **34**, 947–955. Transverse vibration of triangular plates using characteristic orthogonal polynomials in two variables.
- 4. D. J. GORMAN 1983 Journal of Sound and Vibration 89, 107–118. A highly accurate analytical solution for free vibration analysis of simply supported right triangular plates.
- 5. D. J. GORMAN 1986 *Journal of Sound and Vibration* **106**, 419–431. Free vibration analysis of right triangular plates with combinations of clamped–simply supported boundary conditions.
- 6. D. J. GORMAN 1987 Journal of Sound and Vibration 112, 173–176. A modified superposition method for the free vibration analysis of right triangular plates.
- 7. D. J. GORMAN 1989 Journal of Sound and Vibration 113, 115–125. Accurate free vibration analysis of right triangular plates with one free edge.
- 8. D. J. GORMAN 1982 Free Vibration Analysis of Rectangular Plates. Amsterdam: Elsevier.
- 9. R. B. BHAT 1987 *Journal of Sound and Vibration* 114, 65–71. Flexural vibration of polygonal plates using characteristic orthogonal polynomials in two variables.
- 10. C. S. KIM and S. M. DICKINSON 1990 *Journal of Sound and Vibration* 141, 291–311. The free flexural vibration of right triangular isotropic and orthotropic plates.
- 11. K. Y. LAM, K. M. LIEW and S. T. CHOW 1990 *International Journal of Mechanical Sciences* **32**, 455–464. Free vibration analysis of isotropic and orthotropic triangular plates.
- 12. S. STRAND 1988 User Manual, Reference Guide. Ultimo, NSW, Australia, G & D Computing.
- 13. R. M. CHRISTENSEN 1963 American Institute of Aeronautics and Astronautics Journal 1, 1790–1795. Vibration of 45° right triangular cantilever plate by gridwork method.
- P. N. GUSTAFSON, W. F. STOKEY and C. F. ZOROWSKI 1953 Journal of the Aeronautical Sciences 20, 331–337. An experimental study of natural vibrations of cantilevered triangular plates.
- 15. J. R. KUTTLER and V. G. SIGILLITO 1981 *Journal of Sound and Vibration* 78, 585–590. Upper and lower bounds for frequencies of trapezoidal and triangular plates.
- G. R. COWPER, E. KOSKO, G. M. LINDBERG and M. D. OLSON 1969 American Institute of Aeronautics and Astronautics Journal 7, 1957–1965. Static and dynamic applications of a high-precision triangular plate bending element.
- 17. K. M. LIEW 1992 Journal of Engineering Mechanics 118, 1783–1794. Response of plates of arbitrarily shape subject to static loading.

- 18. K. M. LIEW, K. Y. LAM and S. T. CHOW 1989 *Composite Structures* 13, 123–132. Study on flexural vibration of triangular composite plates influenced by fibre orientation.
- 19. R. D. COOK, D. S. MALKUS and M. E. PLESHA 1989 Concepts and applications of finite element analysis. Third edition. New York: John Wiley & Sons.
- 20. O. Č. ZIENKIEWICZ and R. L. TAILOR 1991 *The Finite Element Method*. McGraw-Hill Book Company (U.K.).
- 21. J. H. WILKINSON 1965 The Algebraic Eigenvalue Problem. Oxford: University Press.
- 22. K.-J. BATHE and E. L. WILSON 1976 Numerical Methods in Finite Element Analysis. Englewood Cliffs, NJ: Prentice-Hall.